Constrained Kinematic Fitting for a Top Quark Mass Determination in the Electron + Jets Channel at ATLAS

Diploma Thesis
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Abstract

For the measurement of top quark properties a high purity of the selected events is indispensable. From this perspective the most promising $t\bar{t}$-decay channel is the $e + \text{jets}$ channel, e.g. $t\bar{t} \rightarrow W^+ b W^- b \rightarrow q\bar{q}' b \ell \nu b$. In the current standard reconstruction method, $p_{t}^{\text{max}}$, the jet triplet with the highest transverse momentum is used to reconstruct the properties of the hadronically decaying top quark.

The prospects for a kinematic fitting approach with constraints on the $W^\pm$ boson mass and on the difference of the masses of the two top quarks are presented. The method is tested with Monte Carlo events simulated at next-to-leading order with MC@NLO. Results are obtained, both at the level of partons (parton level) as well as for objects obtained after detector simulation and reconstruction (detector level). At detector level additional contributions from $W + n \text{jets}$ and multijet background events are investigated. The selection efficiency and the improvement on top quark mass measurements are reviewed. Also the prospects for a b jet identification based on the selection presented in this thesis is laid out.
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Overview

In this thesis the method of kinematic fitting with constraints for the selection of e + jets $t\bar{t}$ events at the ATLAS detector is investigated. The work is presented in six chapters with the following content:

Chapter 1: Standard Model of Particle Physics
The Standard Model is presented shortly as well as the LHC and the ATLAS detector. Finally, a brief review of top quark physics is given.

Chapter 2: Monte Carlo Samples and Event Selection Procedure
The signal process as well as the background processes are discussed and the used Monte Carlo samples are introduced. Also different possibilities for an event selection are presented.

Chapter 3: Kinematic Fitting with Constraints
After a short introduction on $\chi^2$ fitting the method of kinematic fitting with constraints is presented and its implementation within this work is discussed.

Chapter 4: Method Validation at Parton Level
The method is validated on a parton level sample with gaussian smearing and compared to the present standard selection method within ATLAS, the so called $p_T^{\text{max}}$ method. Additionally, different technical issues of the method and possible biases are investigated.

Chapter 5: Method Application at Detector Level
The method is applied to a Monte Carlo sample after simulation of the full detector response. The performance is tested on signal as well as on background processes and is compared with the performance of the $p_T^{\text{max}}$ method.

Throughout this thesis natural units $c = \hbar = 1$ are used.
1. Introduction

1.1. Standard Model of Particle Physics

The Standard Model of particle physics describes the properties of the elementary matter constituents as well as the strong, weak and electromagnetic interaction. The elementary matter constituents are spin 1/2 fermions and form three generations of leptons and quarks (columns in Figure 1.1). Vector bosons carry integer spin and mediate the interactions. Additionally, anti-fermions and anti-bosons exhibit opposite values for all non-zero quantum numbers, apart from mass and spin. Up to now, the Standard Model is in good agreement with experimental data. Only a few shortcomings, like the extreme fine-tuning of its parameters, are known. A short introduction will be given in the following paragraphs; confer to [1] for further details.

![Table of Standard Model Constituents](image)

Figure 1.1.: Schematic table displaying the constituents of the Standard Model. Also shown are the electric charges and the masses [2,3].

The Standard Model is a quantum field theory based on the gauge symmetry group SU(3)×SU(2)×U(1). All vector bosons belong to one of the constituting gauge symmetry groups. The amount of gauge symmetry charge determines the strength of the coupling.

Best studied are the electromagnetic and the weak interactions which are combined into the electroweak interaction [4] and are represented by SU(2)×U(1). Their bosons are the
photon ($\gamma$), the $Z^0$- and the $W^\pm$ bosons. Except for the electrically neutral neutrinos ($\nu$) all fermions as well as the $W^\pm$ bosons themselves interact electromagnetically. For the weak aspect of the interaction the charge distribution is more intricate, as only the left-handed\(^1\) fermions and the right-handed anti-fermions carry weak charge.

The gauge group SU(3) represents the strong interaction, which is mediated by gluons. The respective gauge symmetry charge is called colour. Besides the quarks, the gluons themselves carry colour charge. Due to the thereby possible selfinteraction of the gluons, objects with colour charge are confined to colourless objects—so called hadrons—and are only asymptotically free. Thus it is difficult to probe them individually.

In the description of the electroweak interaction, one drawback of the Standard Model was elided: It is not possible to introduce fermion masses in a simple way. This is because left-handed particles are organised in doublets while right-handed particles are arranged in singlets. Ad-hoc mass terms would break the experimentally well tested symmetry strongly.

To solve this drawback the Higgs-mechanism\(^6, 7\) is introduced into the Standard Model. It incorporates a spontaneous symmetry breaking scalar field, which allows to introduce Yukawa coupling induced masses of the fermions. Additionally, it leads to masses of the $W^\pm$ boson and the $Z^0$ boson. If this mechanism is the one realised in nature, there should exist a manifestation of the Higgs-field—the Higgs boson. But, neither was a Standard Model Higgs boson found so far, nor is a precise prediction for its mass offered by theory within the Standard Model. The best indirect constraint on its

\(^{1}\)A particle is called left-handed if the projection of the spin is orientated opposite to the momentum. For right-handed particles both are parallel. While massless particles can only be right or left-handed, massive particles can be boosted to a Lorentz frame where they exhibit the other handedness. However, their coupling is then suppressed by their mass.
large Hadron Collider mass arises from the top quark mass in combination with the $W^\pm$ boson mass (cf. Figure 1.2). The Higgs-mechanism described above is just the simplest form; more than one scalar field or further symmetries could be realised in nature, e.g. supersymmetry [8], which would require at least five Higgs bosons.

1.2. Large Hadron Collider

The Large Hadron Collider (LHC) [9] is a ring collider with 27 km circumference. It was constructed in the former Large Electron-Positron Collider (LEP) tunnel at the CERN laboratory near Geneva. The main goals of the LHC are to find the Higgs boson, to further test the Standard Model and to give indications for physics beyond the Standard Model. First collision data with 3.5 TeV proton energy are planned to be taken from end of 2009. Later in 2010, the centre of mass energy shall be raised to 5 TeV per proton [10].

Figure 1.3.: Sketch of the LHC ring and the main detectors. The two small one, TOTEM and LHCf, adjacent to CMS and ATLAS, are not shown [11].

Four collision points around the ring are equipped with at least one detector each. ATLAS [12] and CMS [13] are designed for a wide range of physics, the other four experiments are devoted to special physics topics. The LHCb [14] experiment investigates the physics of bottom quarks to promote the knowledge of the observed matter-antimatter asymmetry in the universe and the ALICE [15] experiment is dedicated to collisions of heavy ions. The two small experiments investigate the total cross section, elastic scattering and diffractive processes of proton-proton collisions (TOTEM [16]), or deliver a measurement of photons and neutral pions in the very forward scattering region of proton-proton-interactions (LHCf [17]).

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The LHC is designed to accelerate bunches of about $10^{11}$ protons up to 7 TeV per proton, leading to a proton-proton centre of mass energy of 14 TeV. Due to the substructure of the proton only a fraction of the energy contributes to the hard interaction. When running at design luminosity, the LHC will collect about 100 fb$^{-1}$ [18] of data per year, which corresponds to $8 \cdot 10^7 \, t\bar{t}$ events, assuming a cross section of 800 pb [19]. Due to lower beam energies in the beginning and the initially much lower luminosities, the first year of LHC running is expected to yield around 160 000 $t\bar{t}$ events.

1.3. The ATLAS Detector

As one of the multi-purpose experiments, ATLAS is designed to detect as many physics signatures as possible. Besides shedding light on physics beyond the Standard Model, the other central goal of the ATLAS programme is the prominent hunt for the (Standard Model) Higgs boson. Since current experimental limits and theoretical bounds allow a mass range of 114 to 160 GeV and 170 to 200 GeV for the Standard Model Higgs boson [20], it is detectable for ATLAS—if existent—with the predicted properties [18]. Another major topic is the composition of the universe. Until today only about 5 % are understood. A possible explanation for roughly another 25 % could be the lightest supersymmetric particle (LSP), if stable, as predicted by the minimal supersymmetric extension of the Standard Model (MSSM). Finally, the precise measurement of parameters within the Standard Model is an important programme at ATLAS. The LHC provides abundant bottom- and top quarks to study them in great detail. Especially the top quark [21,22] has many interesting features yet only measured with low precision or still only predicted by theory (cf. [23]). Further details on top quark-physics will be given in Section 1.4.

![Figure 1.4.: Sketch of the ATLAS detector and its main components [11].](image)

The ATLAS detector is a 44 m long colossus of 7000 t with a diameter of 25 m [12]. A schematic sketch is shown in Figure 1.4. The high luminosity of the LHC requires radiation hardness of all detector components as well as extremely fast readout electronics. In order to distinguish individual interactions within one bunch crossing, the detector
has a high granularity. Also, the high data rates demand a sophisticated trigger system, since it is crucial to lose as little event information as possible, but not everything can be stored. Thus, ATLAS is designed to be as hermetic as achievable and for example is able to measure the energies of particles up to a pseudorapidity\(^4\) of |\(\eta\)| = 4.9. Further out are only the forward detectors, which are mainly used to monitor the luminosity.

\(\eta := -\ln \tan(\vartheta/2)\).

\(\theta\) are objects that arise from hadronisation of colour charged objects and are reconstructed by the software framework of ATLAS. b Jets are induced by a bottom quark.

Like other colliding-beam-experiments ATLAS has a layout like an onion, in which each layer is constructed to address different tasks. It is subdivided into three functional units (from inside to outside):

- The Inner Detector
- The Calorimeter System
- The Muon System

The Inner Detector is located at the centre of the detector, closest to the beam pipe, and is used to determine the trajectories of charged particles with high accuracy. Its three major parts are the Pixel Detector, the Semiconductor Tracker and the Transition Radiation Tracker (confer to Figure 1.5). The Pixel Detector, which is the innermost layer, has the highest spatial resolution and is crucial for the identification of b jets\(^5\), since it allows secondary vertex identification. It is surrounded by the Semiconductor Tracker. Because the density of trajectories is much lower at this distance to the primary vertex, it is strip rather than pixel based. The outermost Transition Radiation Tracker is

\(^4\)The ATLAS coordinate frame is spanned by \(x\), \(y\), and \(z\). The point (0,0,0) is located in the centre of the detector. The \(x\) axis points to the centre of the ring and the \(y\) axis upwards; righthandedness defines the \(z\) axis, which goes along the beam pipe. Alternatively the notation with the azimuth angle \(\phi\) and the polar angle \(\vartheta\) is used. The angle \(\phi = 0\) points to the centre of the ring and \(\vartheta = 0\) points towards positive \(z\). The pseudorapidity \(\eta\) is defined by \(\eta := -\ln \tan(\vartheta/2)\).

\(^5\)Jets are objects that arise from hadronisation of colour charged objects and are reconstructed by the software framework of ATLAS. b Jets are induced by a bottom quark.
based on straw tubes, fibres (barrel) and foils (end-caps) and serves both as tracker and as particle identification device. While the straw tubes increase the hit multiplicity of the trajectory, the fibres and foils evoke transition radiation, allowing the discrimination of electrons and pions. The whole Inner Detector is embedded into a 2 T solenoid magnet to bend the trajectories for momentum determination. Further details on the Inner Detector can be found in chapter 4 of [12].

The Calorimeter System is used to measure particle energies. It consists of two major parts: The accordion shaped electromagnetic calorimeter and the hadronic calorimeter. Due to the high atomic number of the absorber material, the first one stops and determines the energy of photons, electrons and low energetic, charged hadrons, while the second one has a large hadronic attenuation length and is used to determine the energy of hadrons. At ATLAS all calorimeter parts are so called sampling calorimeters but they use slightly different technologies. While for the tile barrel and tile extended barrel (see Figure 1.6) a scintillator—polystyrene [24]—as active material and steel as absorber is used, in the other parts liquid argon as active material and lead (electromagnetic part), copper, or tungsten (hadronic part) are used as absorber. Further details on the Calorimeter System can be found in chapter 5 of [12].

With its voluminous, superconducting air-core toroid system and the high precision tracking and trigger chambers, the Muon System marks the dimensions of ATLAS. It is designed to measure the momentum of particles that cross the Calorimeter System. Thus it will detect muons beginning at a momentum of a few GeV up to high energetic ones of about 3 TeV. For the tracking and triggering four distinct technologies are used: While the fast Resistive Plate Chambers and Thin Gap Chambers form the trigger system, the Monitored Drift Tubes and the Cathode Strip Chambers are used to perform the tracking. A more detailed review of the magnets and the different chamber types can be found in chapter 6 of [12].
1.4. Top Quark Physics

With a mass of $173.1 \pm 1.3$ GeV [3], the top quark is by far the heaviest known quark. Its existence was first proposed after the discovery of the $\tau$ lepton [25] and the bottom quark [26,27] in the late 1970’s to complete the 3rd generation of quarks. Many indirect measurements indicated its existence, but it was not discovered before 1995 [21,22]. Since the top quark has a lifetime of only $0.5 \cdot 10^{-24}$ s it should not be able to build bound states before its decay [28]. This opens the possibility to measure the mass of an approximately free quark.

Besides constraining the Standard Model Higgs boson mass indirectly, the huge mass of the top quark might allow for a deeper understanding of the Yukawa-mechanism and the electroweak symmetry breaking. Last but not least there are some unknown or only poorly measured quantum numbers of the top quark. Thus, there is still the chance that the observed resonance is an exotic particle and not the predicted isospin partner of the bottom quark.

![Figure 1.7.](image)

Figure 1.7.: At hadron colliders like Tevatron and LHC the four shown leading order processes produce the $t\bar{t}$ pairs. At LHC gluon fusion processes shown in (a) dominate, while at Tevatron the quark antiquark annihilation processes depicted in (b) dominate.

The easiest access to the top quark is achieved via $t\bar{t}$ pairs, which will be produced at the LHC mainly by gluon fusion ($\approx 90\% @ \sqrt{s}=14$ TeV, Figure 1.7(a)) or by quark-antiquark annihilation ($\approx 10\% @ \sqrt{s}=14$ TeV, Figure 1.7(b)). The discovery of single top quark production via the processes shown in Figure 1.8 was only reported recently by D0 and CDF [29,30]. This channel is especially interesting for studies of the $V_{tb}$ CKM matrix element\(^6\). More details about single top quark events can be found in [23].

Due to the large value of $V_{tb}$, the top quark decays to nearly 100% into a $W^\pm$ boson and a bottom quark. Since all fermions regardless of their colour charge are coupled to the $W^\pm$ boson with the same strength, it decays in leading order in $1/3$ of the cases into a charged lepton $\ell$ and a neutrino $\nu$ and in $2/3$ into a $q\bar{q}$-pair. Considering the decay of the $W^\pm$ boson, there are three decay-channels:

In the all-hadronic channel both $W^\pm$ bosons decay into quarks which evolve into jets. Although its contribution at leading order is $36/81$, the high rate of QCD multijet background events render this channel very demanding—especially in the beginning of

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\(^6\)The squared entries $V_{ik}$ of the unitarian Cabibbo-Kobayashi-Maskawa-Matrix (CKM-Matrix) are a measure for the probability that a quark of flavour $i$ turns into a quark of flavour $k$ by the exchange of a $W^\pm$ boson.
Figure 1.8.: The three classes of single top quark production processes are (a) the $W^*$ process and s-channel production, (b) the $W^\pm$ boson-gluon fusion or t-channel process and (c) $W^\pm$ boson-t production.

the experiment. The other extreme, the *di-leptonic channel*, where both $W^\pm$ bosons decay leptonically, makes up only $9/81$ but is very clean due to the two charged leptons. But here the two unmeasurable neutrinos introduce systematic uncertainties. The most promising channel for early LHC physics is the *lepton + jets* channel. Here one $W^\pm$ boson decays leptonically while the other decays hadronically. Until now signatures containing a $\tau$ lepton were not studied widely, because the identification of $\tau$ leptons is very intricate. Measurements in all three decay channels are important as discrepancies to Standard Model predictions could give a hint for new physics.

For the measurement of top quark properties a high purity of the selected events is indispensable. From this perspective the most promising $t\bar{t}$-decay channel is the *lepton + jets* channel. In the following chapters possibilities for an improvement of the event selection within the $e + jets$ channel—a subclass of the lepton + jets channel— will be discussed. The selected events then can be used for a mass determination.
2. Monte Carlo Samples and Event Selection Procedure

For a measurement of top quark properties a clean sample of $t\bar{t}$ events is needed. Only the $e + \text{jets}$ channel is considered in this thesis. The signature of this channel as well as relevant background processes are presented in the following. Furthermore, the Monte Carlo simulation samples used in Chapter 4 and Chapter 5 are introduced.

2.1. The Electron + Jets Channel

Since the $e + \text{jets}$ channel is robust for low statistics it will be the process of choice for first data. Instantaneously after the decay of the $t\bar{t}$ pair there are in leading order essentially four partons\(^1\)—the two bottom quarks ($b_h$, $b_\ell$) and the two light quarks ($q_1$, $q_2$)—, one electron or positron and a $\nu$ or $\bar{\nu}$. Each of the four partons hadronises into many particles like pions, kaons and due to subsequent decays also into electrons and photons. The calorimeter measures this particle shower and the ATLAS software forms jets out of it. Jets are defined throughout this thesis by a cone\(^2\) with a radius of $\Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} = 0.4$. Due to gluon radiation and imperfect jet reconstruction, it can happen that there are more than four jets in an event. On the other hand, if any two partons are closer to each other than the jet resolution parameter they are merged, leading to less than four jets. In contrast the reconstruction of electrons is straightforward.

\(^1\)The symbol $b_h$ denotes the $b$ jet, which originates from the same top quark as the hadronically decaying $W^\pm$ boson. The corresponding $b$ jet on the leptonic side it is called $b_\ell$. The light quarks are indicated by $q_1$ and $q_2$ (cf. Figure 2.1).

\(^2\)The jet algorithm is Cone4 Tower [31, 32].

![Figure 2.1.: One example Feynman graph for the $e + \text{jets}$ channel.](image-url)
The electron is measured in the Inner Detector and in the electromagnetic calorimeter. Since in the early days of large experiments combined objects may not be reliable for analysis, in this thesis the reconstruction of the electron will rely only on the calorimeter part. Nevertheless the Inner Detector is used as a photon veto [18, p. 72 ff]. Furthermore, the electrons are required to exhibit less than 6 GeV energy in a cone-shaped shell of thickness $\Delta R = 0.2$ around them. More details about the isolation criteria can be found in [18, p. 75 ff]. The signature of the unmeasured $\nu$ in the detector is a sizeable missing transverse energy.

The signature for the $e + \text{jets}$ channel consists of at least four jets, exactly one isolated electron and missing transverse energy. Since the correspondences between jets and partons are not known, there is ambiguity for the analysis of the signal sample.

Two kinds of Monte Carlo Samples are used. While the parton level sample is an approximation of an ideal, thereby determining the optimal performance of the method, the full simulation sample aims at the closest possible approximation of reality.

### 2.1.1. Parton Level Sample

A set of 400 000 Monte Carlo $t\bar{t}$ events was produced using MC@NLO in version 3.4 [33,34] with the configuration listed in appendix B. The hard process of $t\bar{t}$ production is calculated at next-to-leading-order (NLO), so that diagrams that produce one additional parton in the final state are included at matrix element level. In this version of MC@NLO, the mass distributions of the $W^\pm$ bosons and of the top quarks are simulated with their respective widths and follow Breit-Wigner distributions. The top quark mass was simulated at 170.9 GeV [35] with width 1.4 GeV. The respective values for the $W^\pm$ boson are $80.398$ GeV and $2.14$ GeV [2]. All events were decayed into the $e + \text{jets}$ channel.

Due to the calculation at NLO the Monte Carlo events are weighted and these weights can be negative as well. This can lead to a negative sum of weights for special phase

![Figure 2.2](image-url)

Figure 2.2: The distribution of (a) the pseudorapidities $\eta$ and transverse momenta for the parton level sample objects. The distribution for $b_h$ and $b_\ell$ are identical.
space regions and thus meaningless results there. Hence a sufficient number of events has to be produced. For this sample the sum of weights is 302 788. The weighted sum of events is called weighted events in the following.

While the $\eta$ distributions for all objects are quite similar (cf. Figure 2.2(a)) the distributions of the transverse momenta differ for the four different species. The on average bottom quarks have the highest transverse momentum since they originate from the initial top quark decay. In contrast the light quarks, the electron, and the $\nu$ can at most obtain the part of the energy, which was carried by the $W^\pm$ bosons. Furthermore, within the decay of the top quark, the large mismatch of the $W^\pm$ boson mass and the bottom quark mass leads kinematically to a bigger amount of the energy for the bottom quark. The difference between the electron and the light quarks is due to the helicity of the $\nu$, which leads to a harder $\nu$ and a softer electron. It has been verified that the distributions in $\phi$ are isotropic for all objects.

### 2.1.2. Full Simulation Sample

The $t\bar{t}$ full simulation sample has been generated with MC@NLO version 3.1. As parton density functions the set CTEQ6.m was used. For all full simulation samples the HERWIG programme [36] was used for fragmentation and hadronisation and the Jimmy library [37] for the underlying event simulation$^3$. The response of the detector was simulated with the GEANT4 code [38]. According to the former plans for LHC start-up, the samples were simulated with a proton proton centre of mass energy of 10 TeV.

![Figure 2.3](image_url)

**Figure 2.3.** (a) Number of events for the different decay channels. The black histogram incorporates all events after preselection. For the red histogram exactly four reconstructed jets and exactly one reconstructed electron is demanded. (b) Multiplicity of reconstructed jets (black) for the $e + jets$ channel. In the red histogram exactly one reconstructed electron is demanded in addition.

$^3$The terminus underlying event labels the impact of partons of the hard interacting protons which do not take part in the hard interaction. These are often called spectator partons.
Here, the simulated mass of the top quark is 172.5 GeV and the mass of the $W^\pm$ boson is 80.403 GeV and its width 2.141 GeV. In contrast to the parton level sample, for which MC@NLO version 3.4 was used, this sample does not incorporate a Breit-Wigner distribution for the top quark mass, while it is present for the $W^\pm$ boson. In addition to the $e^+\text{jets}$ channel all other decay channels, except the all-hadronic one, were simulated. These other channels are treated as background in the following.

The sample contains 87,275 weighted events, where all reconstructed objects exhibit a transverse momentum above 20 GeV and a pseudorapidity between $-2.5$ and 2.5. Additionally, at least one reconstructed electron must be present and jets, which are closer than 0.15 in $\Delta R$ to an electron are removed\(^4\). The contributions of the different decay channels after this preselection are shown in Figure 2.3(a). Also shown are the corresponding numbers after demanding exactly four reconstructed jets plus exactly one reconstructed electron. The main focus of this work is on the $e^+\text{jets}$ channel; the impact of the other channels is discussed in Chapter 5. Since at least one reconstructed electron was present in the beginning, the additional requirement for exactly one electron is only important for the electron part of the di-leptonic channel.

Figure 2.3(b) shows the multiplicity of reconstructed jets for the $e^+\text{jets}$ channel. In about one third of the events only three or less jets are reconstructed and thus cannot be used. For 35,547 events exactly one reconstructed electron is present and four or more jets are reconstructed.

**Matching of top quark decay products to reconstructed objects**

For some studies, like the determination of the resolution it is necessary to have a correspondence (i.e. matching) between a generator level object produced in the hard interaction and its reconstructed object. Thus, for each parton and the electron within $\Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} < 0.2$ a reconstructed object is searched for. No energy requirements are imposed. There are 5% of the events, where a reconstructed jet is matched to two partons. These are discarded.

The distance of the reconstructed object to its generator level correspondence is shown in Figure 2.4 for the different partons. The difference between the distributions of the two light quarks is caused by the fact that they are $p_t$ ordered in the full simulation sample. For $b_h$ and $b_\ell$ their environment is decisive. To illustrate this, the distance in $\Delta R$ of the reconstructed $b_h$ and $b_\ell$ to the second nearest parton is plotted in Figure 2.5 against the reconstructed transverse momentum of $b_h$ and $b_\ell$. For small transverse momenta the event has a more spherical shape and the distances of the bottom quarks to their second nearest neighbours are similar on both sides of the event. For higher transverse momenta the picture changes considerably on the side of $b_h$ as the distances decrease due to the stronger boosts of the jets.

As the jet reconstruction is a complex and imperfect procedure, not all jets can be matched. The matching efficiencies for $b_h$ and $b_\ell$ are about 80%. In the case of the light quarks, the one with the higher transverse momentum $q_1$ can be matched in $(76.8\pm0.2)\%$ of the cases, while the second light quark $q_2$ has a matching efficiency of $(71.2\pm0.2)\%$. The differences are due to the showering properties depending on parton type and energy.

\(^4\)In ATLAS each electron is also treated as a jet. Since these jets would disturb the analysis they are removed.
Figure 2.4.: The distance in $\Delta R$ of the matched jets to their respective parton.

Figure 2.5.: The distance in $\Delta R$ for (a) $b_h$ and (b) $b_\ell$ of the matched jets to the second nearest parton versus the transverse momentum of the reconstructed jet. The red histogram denotes the mean and standard deviation of the distribution per bin.
Table 2.1.: Weighted numbers of events for the full simulation signal sample. Subdivided by number of matched jets versus the number of reconstructed jets.

<table>
<thead>
<tr>
<th>Reconstructed jets</th>
<th>Matched jets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>&gt;6</td>
<td>-3</td>
</tr>
<tr>
<td>∑</td>
<td>98</td>
</tr>
</tbody>
</table>

For electrons the distance between simulated and reconstructed objects is much smaller than for jets. Thus they can always be matched in the used sample.

In total there are 11,664 events where all objects have been matched and exactly one electron is present. Itemised by the multiplicity of reconstructed jets this yields the weighted numbers of events quoted in Table 2.1.

The negative entry for more than six reconstructed jets reflects the occurrence of events with negative event weights and thus renders this phase space meaningless.

Energy Resolution Studies

To perform the kinematic fit, the energy resolution of the jets has to be determined. A Gaussian distributed resolution is assumed:

\[
\frac{\sigma(E)}{E} = \frac{\alpha}{\sqrt{E/\text{GeV}}} \quad \text{or} \quad \frac{\sigma(E)}{E} \sqrt{E/\text{GeV}} = \alpha,
\]  

(2.1)

where \(E\) is the energy of the parton, \(\sigma(E) = E_{\text{reco}} - E\) denotes the deviation of the parton energy with respect to the reconstructed energy, and \(\alpha\) is its respective resolution. The aim is a mass measurement and not a determination of the calorimeter performance, consequently the parton energy is used instead of the simulated jet energy. For the derivation of \(\alpha\) in Equation 2.1, the corresponding distributions are fitted as shown in Figure 2.6. All events are used where all objects are matched (sample \(4+\text{Jms}^5\)). For the light jets and electrons the assumed Gaussian shape of the resolution is adequate and yields values for \(\alpha\) of \((114 \pm 0.8)\%\) for the light jets and \((21 \pm 0.3)\%\) for the electrons. For the bottom quarks, presumably semileptonic \(b\) decays, due to the not measured neutrinos, lead to the pronounced tail on the lower side of the distribution in Figure 2.6(b). Thus for bottom quarks a fit over the full range is not a good approximation of the distribution. A fit of the kernel describes the distribution better, but on the other hand only accounts for roughly three fourths of the events. Also it shows that the right shoulder of the distribution is perfectly Gaussian. Since the selection efficiency does not depend strongly on the exact value of \(\alpha\), as is shown in Section 4.2.3, the RMS—\((148 \pm 0.6)\%\)—of the distribution is used as an estimate for the resolution. In the future, semileptonic decays could be identified by searching for charged leptons within jets. The so found jets can either be removed to avoid the poorly measured part of the distribution, or be used as

\(^5\)A list of the used Monte Carlo samples can be found in Table 2.4.
Figure 2.6: The normalised deviation $\sigma(E)/\sqrt{E}$ of the reconstructed jet energy from the parton energy for (a) the light jets, (b) the b jets, and (c) the electrons. For all three distributions the result of a Gaussian fit for the full range is shown in blue. For the b jets a fit within -2 and 2 is shown additionally in red. (d) Resolution of the top quark mass after simulation of the detector response fitted with a Gaussian. The dotted green line indicates the simulated mass. All distribution are based on all matched events with four or more jets.
Monte Carlo Samples and Event Selection Procedure

identified b jets. The impact of b jet identification is discussed in Section 4.1.5. More on the classification efforts for b jets within the ATLAS collaboration can be found in [18].

Due to not yet fully calibrated jets, the distributions are not centred at zero. The size of the shifts scales with the complexity of the objects. While for well defined electrons a rather small shift of $-0.08 \pm 0.003$ is observed, the light jets yield $-0.32 \pm 0.007$. Finally, the b jets have a shift of $-0.97 \pm 0.009$ mainly because of the semileptonic decays.

These offsets are also reflected in the mass of the reconstructed top quark. In Figure 2.6(d) the top quark mass is derived from sample 4+Jms and the mean of the peak is shifted to $160.7 \pm 0.2$ GeV. Thus the simulated top quark mass (dotted green line) is underestimated. This mainly reflects that the jets are not jet fully calibrated. The choice of the jet algorithm is also known to influence the reconstructed top mass value [39]. Furthermore, the resolution on the mass of the hadronically decaying top quark, which is simulated without a width, is $15.1 \pm 0.2$ GeV.

2.2. Background Processes

In a real data sample, in addition to the signal channel there are processes that mimic it. They are classified as physics background if the signature is identical to the one of the signal channel or as instrumental background if a shortcoming in the measurement or reconstruction is needed to mimic the signal.

Besides some, at least for low luminosities, less important overlays such as the underlying event and minimum bias events\(^6\) the lepton + jets channel suffers mostly from W + n jets and multijet background, defined in the following.

2.2.1. W + n Jets Process and Monte Carlo Sample

The most important physics background is W + n jets background with $n \geq 4$, for which example Feynman diagrams are shown in Figure 2.7. If the $W^\pm$ boson decays leptonically the signature perfectly matches the one of the lepton + jets channel. Since each additional coloured object introduces an additional factor of the strong coupling constant $\alpha_s$, with $\alpha_s(m_{Z0}) = 0.1184 \pm 0.0007$ [40], and four jets are needed to mistake the event with a $t\bar{t}$ lepton + jets event, relevant W + n jets backgrounds are suppressed by at least $\alpha_s^4$.

\(^6\)Minimum bias reactions subsume soft proton-proton interactions. Especially for low luminosities they are not worrysome.

![Figure 2.7: Three example Feynman diagrams for the $W + 4$ jets background processes](image-url)
Here, $W + n$ jets background processes for $n = 4$ and $n = 5$ are considered, where the $W^\pm$ boson decays into an electron or positron and a $\nu$ or $\bar{\nu}$. The Monte Carlo samples were produced with the ALPGEN [41] generator. The numbers of available events are given together with all other backgrounds in Table 2.3.

### 2.2.2. Multijet Background

Multijet events arise mainly from higher order QCD processes and are suppressed, like the $W + n$ jets background events, by at least $\alpha_s^4$. An additional suppression of this background class comes from the requirement of a high energetic, isolated electron and a significant amount of missing transverse energy in the preselection. These electrons can arise from two sources: The decay of hadrons containing bottom quarks or charmed quarks, heavy hadrons, or misidentification of a jet as an electron. Due to the fact, that the reconstruction has to be incorrect this type of background is instrumental. Due to the large cross section, multijet background has a huge rate. Thus even with the high suppression it needs to be dealt with. It can only be determined reliably from data as its Monte Carlo simulation suffers from large uncertainties.

As the total cross section is about 6 mb [42] an inclusive Monte Carlo production is not feasible. Therefore, the choice has been made to simulate the samples in slices of the leading parton $p_t$. Table 2.2 shows the $p_t$ range of the leading parton for the different slices.

Each slice is built up of ALPGEN QCD four (N4) and five (N5) parton samples. Other parton multiplicities are not considered here. Due to CPU limitations a full simulation of the J2 and J3 samples is not feasible. Therefore, on particle level\(^7\) three jets with a transverse momentum above 35 GeV and one additional one above 17 GeV are required. If this is not fulfilled, the event is discarded and does not enter the detector simulation. Since these requirements are stricter than the ones used in the preselection, these samples are only used with the strict phase space requirements described in the following section. Further details on the multijet background samples can be found in [42]. In the following, this multijet background is called QCD background. The samples are named JXNY, where X denotes the $p_t$ slice and Y the number of partons.

### 2.3. Strict Phase Space Requirements for Selection

To reduce background, additional phase space requirements are imposed, which will be called strict. In this analysis the ATLAS top group standard requirements are used [43]. Additional to the preselection, three jets are required to have $p_t > 40$ GeV and the

\(^7\)That is after fragmentation and hadronisation but before the simulation of the detector response.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$p_t$ range [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>J2</td>
<td>35-70</td>
</tr>
<tr>
<td>J3</td>
<td>70-140</td>
</tr>
<tr>
<td>J4</td>
<td>140-280</td>
</tr>
<tr>
<td>J5</td>
<td>$&gt;280$</td>
</tr>
</tbody>
</table>
missing transverse energy has to be above 20 GeV. Since jets, coming from QCD and W + n jets background, typically have far smaller transverse momenta, these backgrounds are reduced substantially by these requirements. Besides the electron induced jet removal described in the preselection, the requirement for exactly one isolated electron reduces the impact of QCD events further, since fake electrons as well as electrons from heavy-hadron decays are rarely isolated [44]. For the signal 20% of the events fulfil the strict phase space requirements in combination with the requirement for exactly four reconstructed jets and one reconstructed electron.

The W + n jets background can be reduced further by using additional discriminating observables like $H_t$ (scalar sum of transverse energy), applanarity (measure for the orthogonal momentum component with respect to the event plane), and others [45]. The possibilities to diminish this background via constrained fitting are laid out in Chapter 5. Finally, b jet identification is well suited to discriminate background events, since they typically lack b jets.

Requiring as above exactly one electron and the strict phase space requirements, the samples have the compositions given in Table 2.3, where the stated uncertainties in Table 2.3 are statistical efficiency uncertainties. Due to the extremely low efficiencies for the QCD events their uncertainties were calculated according to [46]. Although millions of events where generated in the beginning, the number of remaining events are only a poor guess of the real background. Furthermore, the generated QCD events only correspond to a luminosity of about 10 pb$^{-1}$. The QCD samples, where no events are left will be neglected in the following, although their uncertainties would allow for a contribution.

The weighted numbers of events are given in Table 2.4 for all used subsamples.
Table 2.4.: Nomenclature of used signal Monte Carlo subsamples. a: all events after preselection s: strict phase space requirements applied (Section 2.3), and m: stands for matched.

<table>
<thead>
<tr>
<th>Name</th>
<th>Matched</th>
<th>$N_{\text{jet}}$</th>
<th># of weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Preselection</td>
</tr>
<tr>
<td>4Ja(s)</td>
<td>4</td>
<td>18 625</td>
<td>9911</td>
</tr>
<tr>
<td>5Ja(s)</td>
<td>5</td>
<td>11 227</td>
<td>7484</td>
</tr>
<tr>
<td>4+Ja(s)</td>
<td>$&gt;4$</td>
<td>35 547</td>
<td>21 759</td>
</tr>
<tr>
<td>4Jm(s)</td>
<td>x</td>
<td>4</td>
<td>5291</td>
</tr>
<tr>
<td>5Jm(s)</td>
<td>x</td>
<td>5</td>
<td>4082</td>
</tr>
<tr>
<td>4+Jm(s)</td>
<td>$&gt;4$</td>
<td>11 664</td>
<td>6938</td>
</tr>
</tbody>
</table>

2.4. Combinatorial Background

Even in a pure $t \bar{t}$ event sample, one large background remains: combinatorial background. The four correct jets, or at least the three jets which belong to the hadronically decaying top quark, have to be identified. The effect of the combinatorial background is illustrated in Figure 2.8. For the orange histogram the three jets were chosen randomly out of the four correct jets to reconstruct the hadronically decaying top quark and derive its mass. For comparison always the correct triplet was chosen in the green histogram. So the shape of the combinatorial background is much wider and exhibits pronounced tails.

To select the correct jets and assign to them their correct role, various methods are used, denoted by $p_{\text{max}}^t$ [43], $d_{R_{\text{min}}}$ [47], $p_{\text{Balance}}^t$ [18], and kinematic fitting with constraints. The last approach is used in this work. The other methods are shortly introduced for comparison.

$p_{\text{max}}^t$ The $p_{\text{max}}^t$ algorithm selects the jet triplet with the highest transverse momentum. This is based on the assumption that the three jets resulting from the hadronically decaying top quark are directed more likely into the same detector regions than two of them in combination with the $b$ jet belonging to the leptonically decaying $W^\pm$ boson. Although this is a decent assumption at parton level, the situation on reconstructed objects is intricate. This method leads to the correct triplet in about 45% of the cases. The resulting top quark mass is shown in Figure 2.8 (blue triangles). As can be seen the selection results in a broad peak, but is still an improvement with respect to the random selection. This method is the current standard within the ATLAS collaboration.

$d_{R_{\text{min}}}$ For each jet triplet all distances between the three jets are calculated. For each jet triplet the maximum is used to characterize it. Then the minimal triplet is selected. This method is based on a similar assumption of closeness as the $p_{\text{max}}^t$-method and yields similar results. It selects the correct triplet in about 43% and the resulting top quark mass distribution (magenta reversed triangles in Figure 2.8) is slightly broader. However $d_{R_{\text{min}}}$ method performs better for high transverse momenta [47] if compared to the $p_{\text{max}}^t$ method.

$p_{\text{Balance}}^t$ For the $p_{\text{Balance}}^t$ method not only the hadronic side is reconstructed but also the leptonic one. The constraint is that the transverse momentum of the $t \bar{t}$ system...
should be minimal or in some approaches, even more strict exactly zero. While this method worked quite well at Tevatron, where the $t\bar{t}$ system was produced at threshold, the usefulness at ATLAS is limited since the impact of initial (ISR) and final state radiation (FSR) is significantly higher. The triplet is selected correctly in just about 25%, if the missing transverse energy is used for the unmeasured $\nu$ and its $p_{\nu,z}$ component is set to zero; this is comparable to random choosing.

**Kinematic Fitting with Constraints** Besides internal information about the event topology and its kinematics, kinematic fitting with constraints uses external information like the known value of the $W^{\pm}$ boson mass and the assumption that top quark and anti top quark have the same mass. Like the $p_t^{\text{Balance}}$ method it uses the leptonic as well as the hadronic side of the event and selects those jets that best fit the data and the given set of constraints. Not only the high selection efficiency of about 70% for the correct triplet renders this approach attractive, but also the fact, that due to the fitting process the top quark mass resolution is improved. A detailed description of this method is given in the following chapters.

![Figure 2.8](image_url)

**Figure 2.8:** Reconstructed top quark mass based on different selection methods. All histograms are normalised to 100%. For comparison the masses resulting from the correct triplet (green squares) and a random triplet (orange stars) are shown.
3. Kinematic Fitting with Constraints

In the following, the method of kinematic fitting with constraints is introduced. Therefore a short introduction to least squares fitting is given in Section 3.1. In Section 3.2 this concept is extended to include constraints. The possible constraints for the $e + \text{jets}$ channel are reviewed in Section 3.3 and, after a discussion of technical aspects in Section 3.4, their implementation is detailed in Section 3.5.

The method has a long tradition in high energy physics [48,49]. Also in the original top quark discovery a kinematic fit was involved [22]. The quest is to find a formulation, which is apt to the conditions of the LHC and the ATLAS detector.

3.1. Least Squares Fit

The most commonly used fitting method is a least squares fit, introduced by Gauß in 1795 for the calculation of the track of Ceres [50]. To find a set of optimal model parameters, they are optimised with respect to data. The distribution of the measured quantities is assumed to be Gaussian. In mathematical terms:

Let $\vec{m} := (m_i)_{i \in \{1, \ldots, n\}}$ be a vector of $n$ measured values of a quantity and $\vec{\sigma} := (\sigma_i)_{i \in \{1, \ldots, n\}}$ their respective uncertainties. Additionally, let $f(\vec{\alpha}, m_i)$ be a model description based on one measurement and $\vec{\alpha} := (\alpha_i)_{i \in \{1, \ldots, k\}}$ a set of parameters, where $k < n$. Now consider the function:

$$F(\vec{\alpha}, \vec{m}, \vec{\sigma}) = \prod_{i=1}^{n} P_i(\vec{\alpha}, m_i, \sigma_i)$$

where $P_i(\vec{\alpha}, m_i, \sigma_i)$ is the normalised probability for $m_i$ to occur under given $\vec{\alpha}$. With the assumption that $m_i$ is Gaussian distributed this implies

$$P_i(\vec{\alpha}, m_i, \sigma_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp \left\{ -\frac{(m_i - f(\vec{\alpha}, m_i))^2}{2\sigma_i^2} \right\}. $$

Due to the monotonic behaviour of the logarithm, this can be transformed to

$$- \ln F(\vec{\alpha}, \vec{m}, \vec{\sigma}) = \frac{1}{2} \sum_{i=1}^{n} \frac{(m_i - f(\vec{\alpha}, m_i))^2}{\sigma_i^2} + \sum_{i=1}^{n} \ln \sigma_i \sqrt{2\pi} \quad (3.1).$$

By extremising the $\chi^2$ term with respect to $\vec{\alpha}$

$$\nabla_{\vec{\alpha}} \chi^2 = 0$$

\footnote{If $k = n$ the problem is completely determined and no degrees of freedom are left. For $k > n$ the fit is meaningless, because the solution can be accomplished by different parameter sets.}
the optimal set of parameters $\vec{\alpha}$ can be found. By introducing the covariance\(^2\) matrix
\[
V = \begin{pmatrix}
\text{Cov}(m_1, m_1) & \cdots & \text{Cov}(m_1, m_n) \\
\vdots & \ddots & \vdots \\
\text{Cov}(m_n, m_1) & \cdots & \text{Cov}(m_n, m_n)
\end{pmatrix},
\]
\(\chi^2\) in Equation 3.1 can be formulated more convenient as
\[
\chi^2 = 2(\vec{m} - \vec{f})^T V^{-1}(\vec{m} - \vec{f}),
\]
where $\vec{f} := (f(\vec{\alpha}, m_i))_{i \in \{1, \ldots, n\}}$. In this formulation, correlations between the parameters are contained in non-vanishing off-diagonal elements of $V$. Further information on least squares fitting can be found in [51, 52].

### 3.2. Least Squares Fit with Constraints

In the context of constrained fitting every event is treated as an integer entity. Each measurement is interpreted as an estimator for a distinct parameter rather than a measurement of the whole event. This yields $f(\vec{\alpha}, m_i) = \alpha_i$. The symbols $\alpha_i$ and $m_i$ can denote for example jet energies. Since the fit has no degrees of freedom (Number of parameters = Number of measured values), something has to be added to restitute it. One possibility is to introduce further knowledge via constraints, which deliver one degree of freedom each. There are several possibilities to introduce constraints into a fit. Only the two methods that are used in this thesis are described here.

#### 3.2.1. Strict Constraints via Lagrange Multipliers

The first approach assumes constraints that have to be fulfilled exactly. They are introduced via Lagrange multipliers.

Let
\[
(g_i(\vec{m}, \vec{\alpha}))_{i \in \{1, \ldots, l\}} = \vec{0}
\]
be a set of constraint functions of the parameters $\vec{\alpha}$ and the measured quantities $\vec{m}$. Multiplied with the Lagrange multipliers $\vec{\lambda} := (\lambda_i)_{i \in \{1, \ldots, l\}}$ this leads to:
\[
L = \chi^2 - 2\lambda^T \vec{g}
\]
This can be solved by finding the minimum with respect to $\vec{\lambda}$ and $\vec{\alpha}$ simultaneously. If the constraints are not linear functions in $\vec{\alpha}$, the solution of the problem becomes more involved.

Here, a library [53] provided by V. Blobel in combination with ROOT [54] in version 5.18.00 is used for this task. The algorithm of the library solves the problem iteratively. Therefore, it linearises the constraint expressions of Equation 3.2 to
\[
\vec{g}_p + A((\vec{m} - \vec{\alpha'}) - (\vec{m} - \vec{\alpha}_p)) = 0
\]
\(^2\)The covariance $\text{Cov}(m_k, m_l) := \text{E}((m_k - \text{E}(m_k))(m_l - \text{E}(m_l)))$ is equivalent to $\sigma_k^2$ for $m_k = m_l$. The symbol $\text{E}(m_k)$ denotes the expectation value of $m_k$. 


where $\bar{\alpha}_p$ is the set of parameters used in the last iteration, $\bar{\alpha}'$ is the approximation in the current iteration step, $\bar{g}_p := \bar{g}(\bar{\alpha} + (\bar{m} - \bar{\alpha}_p))$, and

$$A := \begin{pmatrix} \frac{\partial g_1}{\partial \alpha_1} & \cdots & \frac{\partial g_1}{\partial \alpha_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_l}{\partial \alpha_1} & \cdots & \frac{\partial g_l}{\partial \alpha_k} \end{pmatrix}_{\bar{\alpha} + (\bar{m} - \bar{\alpha}')}$$

the Jacobian. Combining the derivative of Equation 3.3 and Equation 3.4 into a matrix notation, the problem is reduced to the solution of the equation system

$$\begin{pmatrix} V^{-1} & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} \bar{m} - \bar{\alpha} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ -\bar{g}_p + A(\bar{m} - \bar{\alpha}_p) \end{pmatrix}.$$  

Since after the final iteration the constraint functions $\bar{g}$ are vanishing, a $\chi^2$-like quantity can be calculated by $(\bar{m} - \bar{f})^T V^{-1} (\bar{m} - \bar{f})$. It follows a $\chi^2$-distribution with $n$ degrees of freedom, where $n$ is the number of constraints minus the number of unmeasured quantities. Unmeasured quantities are treated by the library as well and are marked by setting their respective $V^{-1}$ entries to zero. Every unmeasured quantity decrements the degrees of freedom. Apart from some convergence and consistency checks the program iterates until a given precision is reached for the constrained equations. In case of non-convergence it returns an error code.

This formulation allows to introduce strict mass constraints for the top quark and the $W^\pm$ bosons.

### 3.2.2. Loose Constraints

Another possibility is the implementation of loose constraints by adding further $\chi^2$ terms. Let $\bar{C} := (C_i)_{i \in \{1, \ldots, p\}}$ be a vector of constraint values and $\bar{h}(\bar{m}, \bar{\alpha}) := (\bar{h}_i(\bar{m}, \bar{\alpha}))_{i \in \{1, \ldots, p\}}$ a set of models for the quantities $C_i$. Additionally, let $V_{LC} := \text{diag}(\sigma_{i}^2)_{i \in \{1, \ldots, p\}}$ be the matrix of covariances for each $C_i$. This leads to extra terms in the form

$$(\bar{C} - \bar{h}(\bar{m}, \bar{\alpha}))^T V_{LC}^{-1}(\bar{C} - \bar{h}(\bar{m}, \bar{\alpha})).$$

In the limes of infinitesimal small uncertainties these constraints become strict.

Via this formulation uncertainties within a constraint can be dealt with. These arise for example from the Breit-Wigner shape of the $W^\pm$ boson mass distribution.

### 3.3. Possible Constraint Choices for Electron + Jets Events

Of course there are plenty of possible constraints, which can be imposed to a decay process. The $e + \text{jets}$ event topology allows for the following options:

**$W^\pm$ boson mass constraint**  The invariant mass of the system, consisting of the two light jets, as well as the invariant mass of the electron-$\nu$-system can be fixed to the $W^\pm$ boson mass. In the leptonic case this will, due to the unmeasured $\nu$, rather define the $\nu$ itself than sharpen the electron resolution.

---

3 The $\chi^2$ per degree of freedom must be lower than four for the first ten iterations, and below three after ten iterations. Also the fit has to converge within 20 iterations.

4 The term $\text{diag}(x_i)_{i \in \{1, \ldots, n\}}$ denotes an $n \times n$-matrix with $x$, as diagonal entries. All off diagonal entries are zero.
Top Quark mass constraint  In principle, the equivalent would be possible for the invariant mass of the system containing the two light jets and $b_h$ as well as for the invariant mass of the electron, the $\nu$, and $b_\ell$. Here, they would be constrained to the measured value of the top quark mass. Since this would bias the selection for a top quark mass measurement, this option is not used.

Top Quark mass equality constraint  Rather than constraining each invariant mass of the two top quark systems individually to the measured value, they can be constrained to be equal.

No transverse momentum  Also, it can be assumed that the transverse momentum of the $t\bar{t}$ system is zero. While this is a reasonable assumption for the Tevatron, due to ISR and FSR its validity has to be shown for the LHC [18]. Further, the inclusion of this constraint was shown to be not useful [55].

3.4. Technical Considerations

In the following subsections some technical aspects of the constraints and their best realisation are discussed. In Section 3.4.1 the possible permutations are introduced. The treatment of the jets and the electron within the fit is reviewed in Section 3.4.2. The unmeasured $\nu$ is covered in Section 3.4.3. Finally, the accounting of the intrinsic widths of the $W^\pm$ boson mass and the top quark mass difference is discussed in Section 3.4.4.

3.4.1. Possible Permutations

In the case where four jets were reconstructed, the fit has to be performed for 12 permutations, which are given in Table 3.1. Since an exchange of $q_1$ and $q_2$ would neither affect the $W^\pm$ boson mass constraint nor the equal top quark mass constraint these permutations are not distinguished. If there are more reconstructed jets, the additional jet can take up each position and the number of permutations increases.

As can be seen the leptonic part of the $W^\pm$ boson mass constraint cannot be violated, since electrons are not treated as jets.

3.4.2. Treatment of the Jets and the Electron

Since the resolution of the ATLAS calorimeter is considerably better in $\eta$ and $\phi$ than in the energy the directions of the jets and the electron are considered to be exactly measured and only the energies are used as fitting variables. This means that for each step of the fitting procedure the four-vectors $x$ with energy $E$ are transformed to the new energy $E'$ like

$$x' = x \frac{E'}{E_i},$$

with $i = (b_h, b_\ell, q_1, q_2, \ell)$ the $\nu$ is treated slightly different.
### Table 3.1.: List of permutations

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Class</th>
<th>Constraint violated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ok</td>
<td>All assignments are correct</td>
<td>Correct</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b_t \leftrightarrow b_h$</td>
<td>The assignments of the two b jets are interchanged</td>
<td>Incorrect</td>
<td>$m_{W_L}$, $m_{W_R}$, $\Delta m_t$</td>
</tr>
<tr>
<td>$b_h \leftrightarrow q_1$</td>
<td>$b_h$ is interchanged with one light quark</td>
<td>Correct Triplet</td>
<td>x</td>
</tr>
<tr>
<td>$b_h \leftrightarrow q_2$</td>
<td>$b_h$ is interchanged with the other light quark</td>
<td>Correct Triplet</td>
<td>x</td>
</tr>
<tr>
<td>$b_t \leftrightarrow q_1$</td>
<td>$b_t$ is interchanged with one light quark</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b_t \leftrightarrow q_2$</td>
<td>$b_t$ is interchanged with the other light quark</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b_t \rightarrow b_h \rightarrow q_1$</td>
<td>$b_t$, $b_h$, and one light jet are interchanged cyclical</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b \leftrightarrow q$</td>
<td>Each b jet is interchanged with one light jet, $b_h \rightarrow q_1$, $b_h \rightarrow q_2$</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b_t \rightarrow b_h \rightarrow q_2$</td>
<td>$b_t$, $b_h$, and the other light jet are interchanged cyclical</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b' \leftrightarrow q$</td>
<td>As last permutation, but b jets interchanged</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b_h \rightarrow b_t \rightarrow q_1$</td>
<td>$b_t$ and $b_h$ are interchanged and then permuted cyclical with one light jet</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
<tr>
<td>$b_h \rightarrow b_t \rightarrow q_2$</td>
<td>$b_h$ and $b_t$ are interchanged and then permuted cyclical with the other light jet</td>
<td>Incorrect</td>
<td>$m_{W_L}$</td>
</tr>
</tbody>
</table>

### 3.4.3. Treatment of the Neutrino

Since the $\nu$ is unmeasured, its four-vector is undefined. There are several approaches to recover this four-vector. For the $p_x$ and $p_y$ components, the missing transverse energy is taken as an estimate. Although ISR and FSR play a role at the LHC this is a decent approach as shown in Figure 3.1(a). Recovering the $p_{\nu,z}$ component is more difficult. There are two straightforward approaches. In the standard method $p_{\nu,z}$ is derived from the leptonic $W^\pm$ boson mass constraint. The full calculation presented in Appendix A yields:

$$
P_{\nu,z}^{(1,2)} \approx \frac{a}{p_{t,\perp}} p_{t,\perp} + \frac{E_\perp}{p_{t,\perp}} \sqrt{a^2 - p_{t,\perp}^2 (p_{\nu,x}^2 + p_{\nu,y}^2)}
$$

(3.5)

with the abbreviation $a := \frac{m_W^2}{2} + p_{t,x} p_{\nu,x} + p_{t,y} p_{\nu,y}$ and $p_{t,\perp}$ indicating the transverse momentum of the electron. If the discriminant evaluates negative, $p_{\nu,z} = 0$ is taken as a fallback solution. In most of the cases one of the two solutions is a good approximation.
of the 'real' z-component of the momentum as seen in Figure 3.1(c). The problematic cases are often simply discarded [55].

The novel approach makes use of the fact that the pseudorapidity \( \eta \) of the \( \nu \) is distributed nearly Gaussian as shown in Figure 3.1(b) and has a fitted width \( \sigma_\eta \nu \) of about 1.5. Thus, for the neutrino, the fitting variable is either the z-component of its momentum or its pseudorapidity. The \( x \) and \( y \) components of the \( \nu \) momentum are already determined by the other fitted four-vectors, as the \( x \) and \( y \) components of the neutrino momentum need to be recomputed whenever the four-vector of a jet or the electron is changed. To achieve this, the current \( x \) and \( y \) components for all jets, the electron and the missing transverse energy are summed up in each fitting step. The same is done after jet rescaling and the difference plus the old neutrino values result in the new values of the neutrino.

The performance of both approaches is compared in Section 4.1.4.

3.4.4. Loosening of the Constraints

The \( W^\pm \) boson mass as well as the top quark mass follow a Breit-Wigner distribution with width \( \Gamma_W \) and \( \Gamma_t \) respectively. As the actual mass value of the top quark and the anti top quark in each event are independent\(^5\) this shape is preserved also for the \( \Delta m_t \) distribution. As for a \( \chi^2 \) based fit Gaussian uncertainties are needed, the kernels of this distributions are fitted with a Gaussian. This results in \( \sigma_{m_{W,i}} = 6 \) GeV for the \( W^\pm \) boson masses and in \( \sigma_{\Delta m_t} = 5 \) GeV for the top quark mass difference distribution.

As is shown in Section 4.1.3, the inclusion of the finite width of the \( W^\pm \) boson in the fit remains important, even if the detector resolution is taken into account. Hence, the mass constraints are loosened by adding three additional \( \chi^2 \) terms. They allow fluctuations around the pole value within \( \sigma_{m_{W,i}} \) or \( \sigma_{\Delta m_t} \), respectively.

Another approach would be, to use a log likelihood based fit and treat the object shapes generically. But this suffers from the need for a sophisticated balancing of the different terms. Also a Bayesian formulation based on Markov chains is possible and currently investigated in Göttingen [56].

3.5. Building the Fitting Function

Given the considerations above, the following mathematical picture is achieved:

The two strict parts of the \( W^\pm \) boson mass constraints result in

\[
\begin{align*}
\frac{\mathbf{g}_1}{\mathbf{g}_2} &= \sqrt{(p_{q_1} + p_{q_2})^2 - m_{W,1}^2} = 0, \\
\frac{\mathbf{g}_1}{\mathbf{g}_2} &= \sqrt{(p_e + p_\nu)^2 - m_{W,t}^2} = 0,
\end{align*}
\]

and the strict part of the equal top quark mass constraint in

\[
\frac{\mathbf{g}_3}{\Delta m_t} = \sqrt{(p_e + p_\nu + p_b)^2 - \sqrt{(p_{q_1} + p_{q_2} + p_{b_1})^2 - \Delta m_t^2}} = 0,
\]

\(^5\)This is evident from theory as well, since the application of the Feynman rules leads to the integration \( \int dQ^2 dQ'^2 \frac{1}{(Q^2 - m^2) + m'^2} \frac{1}{(Q'^2 - m^2) + m'^2} \), where only \( p = Q + Q' \) is fixed. The symbol \( p \) denotes the four-momentum of the incoming particle(s). The four-vectors of the \( t\bar{t} \) pair are labelled by \( Q \) and \( Q' \).
3.5 Building the Fitting Function

Figure 3.1: (a) The $x$ component of the Monte Carlo simulation neutrino four-momentum drawn against the $x$ component of the missing transverse energy after simulation of the full detector response (sample 4+Ja).
(b) Distribution of the pseudorapidity for the $\nu$ (black) with a fitted Gaussian (red).
(c) For each event of the parton level dataset the simulated $p_{\nu,z}$ is drawn against the solution for which $|p_{\nu,z}^{(1,2)} - p_{\nu,z}|$ is minimal. The cases where no solution could be found for $p_{\nu,z}^{(1,2)}$ show up in the line at zero.
with \( p_{q_1} \) and \( p_{q_2} \) being the four-vectors of the light jets, \( p_{b_1} \) and \( p_{b_2} \) the respective ones of the b jets, and \( p_\ell \) as well as \( p_\nu \) giving the ones of the electron and the \( \nu \), respectively. The terms \( m_{W,h} \) and \( m_{W,\ell} \) stand for the values of the \( W^\pm \) boson masses in this event. Their 'measured' value in terms of the fit is \( 80.398 \pm 0.025 \) GeV [2]. The symbol \( \Delta m_t \) denotes the difference of the top quark and the anti top quark mass. Its 'measured' value is zero.

The loosening terms are integrated into the covariance matrix of the \( \chi^2 \) terms. In the notation of Section 3.1 this is:

\[
\vec{m} = (E_{b_1}, E_{b_2}, E_{q_1}, E_{q_2}, E_\ell, p_{\nu,z}, m_{W,h}, m_{W,\ell}, \Delta m_t) \tag{3.9}
\]

with \( E_i \) the energy of the respective particles and the covariance matrix

\[
V := \text{diag}(\sigma_{E_{b_1}}^2, \sigma_{E_{b_2}}^2, \sigma_{E_{q_1}}^2, \sigma_{E_{q_2}}^2, \sigma_{E_\ell}^2, 0, \sigma_{m_{W,h}}^2, \sigma_{m_{W,\ell}}^2, \sigma_{\Delta m_t}^2) \tag{3.10}
\]

Since the neutrino is unmeasured its assigned uncertainty is zero. If its pseudorapidity is used as fitting variable, \( p_{\nu,z} \) in \( \vec{m} \) is exchanged with \( \eta_\nu \) and zero in \( V \) with \( \sigma_{\eta_\nu}^2 \).

The entire fitting function is then given by:

\[
L = 2(\vec{m} - \vec{\alpha})^T V^{-1} (\vec{m} - \vec{\alpha}) - 2 \vec{\lambda}^T \vec{g} \tag{3.11}
\]

This fitting function is solved for all permutations and all \( \nu \) solutions. The permutation with the smallest \( \chi^2 \) is selected. The performance and habits of this method are discussed in the following chapters.
Starting from a kinematically well defined basis (parton level) the approximation of reality (full simulation) is approached, which will be discussed in Chapter 5. The capability of the kinematic fit as selection method for the reduction of combinatorial background is explored in Section 4.1. After a review of the technical properties in Section 4.2, further use-cases are discussed in Section 4.3. Finally, the improvement on the energy resolution of jets, electrons, as well as on the $W^\pm$ boson and the top quark masses are discussed.

4.1. Method Validation

In the following, the method is validated on the parton level sample. To emulate the resolution of the ATLAS detector the\(^1\) parton and electron four-vectors are smeared with a Gaussian, whose width is $\alpha\sqrt{E/\text{GeV}}$. The symbol $E$ denotes the simulated energy of the object.

The different values of $\alpha$ for electrons, light jets, and b jets are derived from the full simulation sample (cf. Section 2.1.2). These are $\alpha_{u,d,s,c} = 114\%$ for light jets, $\alpha_b = 148\%$ for b jets and $\alpha_\ell = 21\%$ for electrons. Independently an energy loss term is introduced. This accounts for example for a miscalibrated jet energy scale. Again, the values are derived from the full simulation sample. For light quarks, the Gaussian mean is shifted by $-0.32$, for bottom quarks by $-0.97$, and for electrons by $-0.08$.

The selection efficiency is analysed using the standard implementation of the kinematic fit; $p_{\nu,z}$ is used as fitting variable and the widths of the top quark and $W^\pm$ boson are accounted for by loose constraints. The effect of an energy loss is discussed in Section 4.1.2. In Section 4.1.3 the influence of different width hypotheses for the top quark and the $W^\pm$ bosons is covered. The differences between $p_{\nu,z}$ and $\eta_\nu$ based fitting are presented in Section 4.1.4. Finally, additional enhancements of the method like b jet identification, wide mass limits, and $P(\chi^2)$ requirements are investigated in Section 4.1.5.

4.1.1. Distribution of Selected Permutations

To determine the purity and the efficiency all twelve permutations of the jets are fitted. Each permutation is fitted for both $p_{\nu,z}$ solutions or at least for $p_{\nu,z} = 0$, if the discriminant is negative. Out of the converged fits the one with the lowest $\chi^2$ is chosen. Initially, the 12\% of the events, from the MC@NLO sample where an additional parton is present in the final state are taken out and the full phase space of the $t\bar{t}$ decay products is used.

The distribution of the selected permutations is shown in Figure 4.1. The $p_T^{\text{max}}$-selection is given as a benchmark.

\(^1\)Although the pure MC@NLO four-vectors of the partons should not be used for physics measurements [57], they can be used for a validation of the method.
Figure 4.1.: Permutation with the smallest $\chi^2$. The selection of the kinematic fit with loose constraints is represented by the black circles. The blue triangles denote the four possible combinations of the $p_t^{\text{max}}$ method. The green squares indicate the same as the black histogram, but permutations within the selected triplet are not discriminated. All histograms are individually normalised to 100% with respect to the number of converged fits. The convergence rate is given in parentheses. The permutations are defined in Table 3.1.

There are two main differences of the kinematic fit and the $p_t^{\text{max}}$ method. The kinematic fitting approach does not converge in all configurations due to extreme outliers of the Breit-Wigner distributed masses and resolution influences. The dependence on the resolution is studied in Section 4.2.3. For the standard resolution, the fit converges for 98.7% of the events. In contrast, the $p_t^{\text{max}}$ method always yields a jet triplet. The second difference is that the $p_t^{\text{max}}$ method cannot discriminate between different assignments within the hadronically decaying top quark and thus classifies in the four jet case only four different permutations of jets. If the methods are just used for selection, the permutations within the hadronically decaying top quark will be equivalent for a mass determination, but if the altered four-vectors are used for an improvement on the top quark mass resolution, the actual permutation will be important. This effect is investigated in Section 4.3.

For the smeared parton level sample, the kinematic fit assigns all jets correctly for $(56.0 \pm 0.1)$ % of the events. The quoted uncertainty is the statistical efficiency error. For a comparison with the $p_t^{\text{max}}$ method, the histogram is rebined such that permutations within the hadronically decaying top quark are not discriminated (green in Figure 4.1). Taking the rate of convergence into account, the kinematic fit yields an efficiency of $(70.0 \pm 0.1) \times 0.987\% = (69.1 \pm 0.1)\%$. In comparison the $p_t^{\text{max}}$ method yields only $(39.4 \pm 0.1)\%$ efficiency. The most difficult configurations to distinguish for the kinematic fit involve a bad assignment of $b_h$. This reflects two facts: First, these three permutations only violate one constraint—either the hadronic $W^{\pm}$ boson mass constraint or the top quark mass equality constraint—while the others violate two—the hadronic $W^{\pm}$ boson mass constraint and the top quark mass equality constraint. Second, a violation of the
top quark mass equality constraint is less severe than a violation of a $W^\pm$ boson mass
constraint, since the uncertainties on the energies for $b$ jets are larger and more objects
are involved which can counteract. As an example the fit can tune the mass of the other
top quark. In contrast, the $W^\pm$ boson mass constraint is more stringent, since only
two more correlated\(^2\) objects with smaller uncertainties are involved. However, two of
these three permutations only alter the permutation within the hadronically decaying
top quark.

Taking events with an additional parton in the final state into account without treating
the additional parton as an additional jet, changes the selection efficiency of the kinematic
fit by less than one percent. Hence, these events are included.

If the strict phase space requirements are applied, only one third of the events is kept.
While the result of the kinematic fit changes only marginally, the $p_t^{\text{max}}$ method gains
considerably and yields $(47.7 \pm 0.1)\%$. For the reminder of this chapter the phase space
requirements are not imposed if not explicitly stated.

### 4.1.2. Influences of Energy Loss

If in addition to the smearing an energy loss is simulated the convergence rate reduces
to $98.4\%$ and the correct permutation is selected in $(51.3 \pm 0.1)\%$ of the events. So,
as expected, the fit method profits from better calibrated jets. As will be seen in
Section 4.3.2, the method compensates to some extent the energy loss in the fitting
process. A mass analysis can benefit from this, if the altered four-vectors are used and
not only the selection.

### 4.1.3. Strict versus Loosened Constraints

To analyse the influences of the assumed $W^\pm$ boson width $\sigma_{m_{W,i}}$ and the width of the
top quark mass difference distribution $\sigma_{\Delta m_t}$, the parton level dataset with the simulated
Breit-Wigner distributions, is processed with three fit assumptions for the $W^\pm$ boson
width as well as three fit assumptions for the top quark width. The convergence rates
and the purities are shown in Figure 4.2. Width zero corresponds to a strict constraint
for the respective object. All uncertainties are of the order of $10^{-3}$.

If the constraints are strict for both $W^\pm$ boson masses and $\Delta m_t$ (lower left corner
of Figure 4.2), the percentage of correctly selected jets drops by $2.6\%$ absolute to
$(53.4 \pm 0.1)\%$. The convergence rate is $91.9\%$ instead of $98.7\%$. This results in a total
loss of $6.2\%$ absolute, when compared to $(56.0 \pm 0.1) \times 0.987\% = (55.3 \pm 0.1)\%$ for the
standard implementation (upper right corner of Figure 4.2).

The kinematic fitting method is more sensitive to changes of the assumptions on the
$W^\pm$ boson width than on the width of the top quark mass difference. While the normalised
selection efficiency for the correct combination decreases slowly, the convergence rate
drops considerably with decreasing $W^\pm$ boson width hypotheses. In contrast, changing
the top quark width has almost no impact, if the $W^\pm$ boson constraint is relaxed. This is
another manifestation of the fact that the $W^\pm$ boson mass constraint is more restrictive.

\(^2\)About correlations see Section 4.2.2.
Figure 4.2: Each cell represents one combination of $W^\pm$ boson and top quark width assumed in the fit. The numbers give the percentage of events where the correct jet triplet was selected. The colour code indicates the convergence rate.

4.1.4. Influence of Neutrino Treatment

As pointed out in Section 3.4.3 there are two possibilities to treat the neutrino. Figure 4.3 shows the distribution of the selected permutations for the momentum and the pseudorapidity driven approach. The differences between the two methods are rather small. For both methods, the correct permutation is selected in about 55.5% of the events and the convergence rate is about 99%. If the uncertainty assumption for $\eta_\nu$ is reduced to 0.5 instead of 1.5, the convergence rate is reduced to 97.3% and also the selection efficiency goes down to (54.2 ± 0.1) %.

This small change becomes understandable if the $\sigma_{\eta_\nu}$ hypotheses are translated into $\sigma_{p_{\nu,z}}$ hypotheses. Neglecting the uncertainty for $p_{\nu,t}$ this yields

$$\sigma_{p_{\nu,z}} = \sigma_{\eta_\nu} p_{\nu,t} \cosh \eta.$$ (4.1)

For $\sigma_{\eta_\nu} = 1.5$ this corresponds to a rather huge uncertainty assumption for the $\nu$ which does not impose severe restrictions. On the other hand, the probably overestimated knowledge on the $\nu$ expressed by $\sigma_{\eta_\nu} = 0.5$ still allows fluctuations more than twice as large as the ones of the electron.

However the $\eta_\nu$ approach puts all objects on the same footing and avoids solving the quadratic equation for $p_{\nu,z}$. In the following both approaches are investigated in parallel.
Figure 4.3.: Permutation with the lowest $\chi^2$ for the different $\nu$ treatments. Each histogram is normalised separately to 100 %. The black circles show the kinematic fit based on $p_{v,z}$. The kinematic fit with $\eta_\nu$ as fitting variable is drawn in green for $\sigma_{\eta_\nu} = 1.5$ and in blue for $\sigma_{\eta_\nu} = 0.5$. The permutations which yield small contributions and are not discriminated by the $p_{t,\text{max}}$ method are subsumed under the label “rest”.

4.1.5. Additional Enhancements

On top of kinematic fitting there are some possibilities to improve the purity of the selected sample. Their prospects are discussed in the following.

Influence of $b$ Jet Identification

The selection procedure can benefit from $b$ jet identification, since this reduces the number of allowed permutations. For ATLAS, the $b$ jet identification efficiency will be around 60 % or more [18, Chapter: $b$-Tagging]. To determine the influence of $b$ jet identification, these 60 % are used as assumption. The chance to mistake a light jet for a $b$ jet, the mistag rate, is very low and thus neglected.

For every event, each $b$ jet is considered as identified as $b$ jet with a probability of 60 %. There are three possible outcomes: Either none, one, or both $b$ jets are identified. While the first case, does not introduce any changes, the second one excludes six permutations. In the last case, only the first two permutations are left.

The permutation with the lowest $\chi^2$ is shown in Figure 4.4. For the comparison it has to be taken into account that the convergence rate drops by 1.2 % absolute since for some events only permutations that are excluded by $b$ jet identification converged. For the correct permutation this results in $(70.4 \pm 0.1) \times 97.2 \% = 68.4 \%$ efficiency—to be compared to 55.3 %. If one is only interested in the correct triplet, the gain diminishes to 5.1 % absolute, since the permutations where $b_h$ is interchanged with one of the light quarks ($b_h \leftrightarrow q_1$ and $b_h \leftrightarrow q_2$) resolved, but not counted as an improvement.

Of course also the $p_{t,\text{max}}$ method benefits from $b$ jet identification.
Figure 4.4.: Same as Figure 4.3, but for the kinematic fit with different additional enhancements. Each histogram is normalised separately to 100%. While the black circles show the pure kinematic fit, the green squares denote the kinematic fit with b jet identification. The blue histogram illustrates the influence of a top quark mass range limitation from 140 GeV to 200 GeV and the red one a $\chi^2$ probability limit of $P(\chi^2) > 0.15$. The numbers in parentheses give the fraction of selected events with respect to all events in the sample.

**Mass Range Cut**

Due to the fact that the top quark mass is already known to be roughly 170 GeV, a loose limitation of the mass range can be introduced without biasing the mass measurement itself, while improving the selection. If the reconstructed top quark mass resulting from the fit improved four-vectors, is allowed to be within the range of $(170 \pm 30)$ GeV, for $(66.5 \pm 0.1)$% of the events the correct permutation is selected. Taking into account the configurations which do not alter the top quark mass, the kinematic fit has an efficiency of $(83.1 \pm 0.2)$%, however the overall fraction of selected events—the number of converged events which fulfil the requirements divided by all events in this sample—drops to $82.2\%$ resulting in an overall decrease of $11\%$ absolute. This approach helps also when using the $p_{t}^{\text{max}}$ method. In this case, the limit is applied to the reconstructed mass based on the initially reconstructed jets. The $p_{t}^{\text{max}}$ method results in $(62.0 \pm 0.1)$% efficiency on 61.1% of the events passing the mass range limitation.

This enhancement is especially well suited to select a sample with high purity, as they are needed for b jet identification based on $t\bar{t}$ pairs [18, Chapter: b-Tagging Calibration with $t\bar{t}$ Events]. Prospects for this approach are discussed in Section 4.3.1.
4.1 Method Validation

P($\chi^2$) Cut

Another possibility for improvement is to limit the probability of the $\chi^2$ value. As Figure 4.4 shows, the impact is rather small if the limit is placed at $P(\chi^2) > 0.15$. Taking the fraction of selected events of 92.3\% into account, the efficiency of (57.2 ± 0.1)\% drops below the one of the pure kinematic fit. Nevertheless, this requirement can be useful to reduce background as discussed in Chapter 5.

Since the $p_t^{\text{max}}$ method does not yield a quality measure for its selection like the $\chi^2$ probability this possibility is not applicable.

4.1.6. Concluding Remarks

Each of these additional enhancements can be combined with the others. The results for all combinations and the different neutrino implementations are given in Table 4.1 for the smeared sample. The corresponding tables for the energy shifted sample as well as the sample with strict phase space requirements and additional parton exclusion are given in Appendix C.

As can be seen the kinematic fitting approach has a significantly better selection efficiency when compared to the $p_t^{\text{max}}$ method for all configurations investigated.

Table 4.1.: Purities and selection efficiencies for the parton level sample. The numbers are given for both $\nu$ implementations (T) in combination with the different enhancements. The abbreviation KF denotes the kinematic fit, b stands for b jet identification, $P$ for a $P(\chi^2)$ limit of 0.15, and $m_t$ for the top quark mass range limitation of 170 ± 30 GeV. The efficiency of the kinematic fit is the product of the triplet purity times the fraction of selected events (SE). All numbers are in \% and all uncertainties are 0.1\%.

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<tr>
<td>83.8</td>
<td>80.1</td>
<td>87.2</td>
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<td>69.0</td>
<td>45.4</td>
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<tr>
<td>77.1</td>
<td>81.4</td>
<td>88.2</td>
<td>68.1</td>
<td>65.8</td>
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<td>45.4</td>
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<tr>
<td>76.6</td>
<td>80.9</td>
<td>87.8</td>
<td>67.3</td>
<td>65.8</td>
<td>69.0</td>
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</table>
4.2. Technical Studies

In the following sections some technical aspects of the method are described. First, the $\chi^2$-distribution is presented. It follows a discussion of the correlation matrix and a analyses of the influence of the assumed energy resolutions.

4.2.1. $\chi^2$ Probability Distribution

As mentioned in Section 3.2.1 the $\chi^2$ values should follow the theoretical distribution, i.e. a flat $P(\chi^2)$ distribution is expected, if the constraints are strict. Figure 4.5 shows the $\chi^2$ probability distribution for both neutrino treatment implementations for the correct (black, red) as well as for the wrong permutations (green, purple). The slope is due to the fact that the selection process is $\chi^2$-based, and thus there is an enhancement towards lower $\chi^2$ values or larger $\chi^2$ probabilities. The $\chi^2$ probability distribution of the correct permutation without the selection is flatter, in the case of the momentum driven approach (blue) and nearly flat in the case of the pseudorapidity driven approach (turquoise). Overall, the fit based on the pseudorapidity is flatter, since in that case the $\nu$ has an uncertainty assigned and therefore contributes to the $\chi^2$ value. The remaining slope is caused by the fact that the mass distributions are approximated by wide Gaussians while in reality they are Breit-Wigner distributed. That the wrong permutations tend to have lower $\chi^2$ probabilities is intuitive, since they are systematically wrong. But since they are only selected, if they are the best permutation, i.e. have the lowest $\chi^2$, they also show a peak at high $\chi^2$ probabilities.

![Figure 4.5: Distribution of $P(\chi^2)$ for the selected permutations for the momentum driven implementation (black $\equiv$ correct and green $\equiv$ incorrect) and for the pseudorapidity driven approach (red $\equiv$ correct and purple $\equiv$ incorrect), as well as for all correct permutation without respecting the selection (blue and turquoise).](image-url)
Figure 4.6: $P(\chi^2)$ probabilities versus the mass of the hadronically decaying top quark, $m_{t,h}$, split up by permutation class and $\nu$ treatment.
Figure 4.6 depicts the distribution of $\chi^2$ probabilities versus reconstructed top quark masses resulting for both $\nu$ treatments. The distributions are split into the correct permutation (Figure 4.6(a)–(b)) the correct triplet (Figure 4.6(c)–(d)), and the wrong permutations (Figure 4.6(e)–(f)). The wrong permutation and the triplet preserving permutations posses peaks for low $P(\chi^2)$ values. However, due to the relative proportion of the histograms in this region, the discrimination power of the $P(\chi^2)$ limit is rather small. The plots also make the good performance of the mass range limitation understandable, since the distribution for the correct permutation clusters around 170.9 GeV—the simulated value. A comparison between the distributions for $\eta_{\nu}$ and the $p_{\nu,z}$ implementation shows that the latter ones result in a slightly slimmer mass distribution.

### 4.2.2. Parameter Correlations

From the covariance matrix delivered by the used fitting library the correlation coefficients\(^3\) $\rho_{lk}$, can be deduced, which are a measure of the correlation between the $l$-th and the $k$-th variable of the fit. A value of $(-)100\%$ indicates that two variables are fully (anti)correlated. Zero on the other hand stands for uncorrelated variables. Due to the constraints, the correlation of the $l$-th and the $k$-th variable is influenced also by other variables. All correlation coefficients are shown in Figure 4.7(a).

The anticorrelation of the two light jets of $-38.6\%$ is a result of the hadronic $W^\pm$ boson

\(^3\)The definition of the correlation coefficient is $\rho_{lk} := \frac{\text{Cov}(m_l,m_k)}{\sigma_l \sigma_k}$. The covariances $\text{Cov}(m_l,m_k) = \sigma_l$ after the fit are also delivered by the library.
mass constraint. If the fit raises the energy of one light jet it has to lower the energy of the other one, or it will alter the $W^\pm$ boson mass estimator. This second possibility is the reason for the stronger correlation of the light jets with the $W^\pm$ boson mass estimate and explains why the two light jets are not fully anticorrelated, as it is the case for a strict $W^\pm$ boson mass constraint.

The leptonic $W^\pm$ boson constraint stabilises the fit and prohibits that all uncertainties are absorbed by the unmeasured neutrino. This influence can be seen in Figure 4.7(a) in the blue marked region. That these small entries arise from the neutrino can be seen from Figure 4.7(b), where the unmeasured neutrino is hypothetically treated like a measured electron, i.e. with $\sigma_\nu = \sigma_\ell$ and a smeared four-vector. Also, it is evident that the neutrino is the reason for the asymmetry of the correlation coefficients of $E_{b_0}$ and $E_{b_\ell}$ with $m_{W,\ell}$ and $\Delta m_t$.

The main influence region of the equal top quark mass constraint is indicated in red in Figure 4.7(a). The opposite sign in the correlation coefficients of $E_{b_0}$ and $E_{b_\ell}$ with respect to the top quark mass difference arises from the structure of the constraint which is formulated in Equation 3.8 as

$$g_3 = m_{t,\ell} - m_{t,h} - \Delta m_t = \sqrt{(p_\ell + p_\nu + p_b)^2} - \sqrt{(p_q_1 + p_q_2 + p_{b_h})^2} - \Delta m_t \neq 0$$

and not as

$$g_3 = m_{t,h} - m_{t,\ell} - \Delta m_t = 0$$

It has been verified, that treating the $\nu$ via its pseudorapidity does not significantly change the result shown in Figure 4.7(a). This confirms the similarity of the two approaches.

### 4.2.3. Dependence on the Resolution Assumption

As in data Monte Carlo information cannot be used to determine the energy resolution, the fitting method should be robust against misestimated resolutions. As will be shown in this section, for the aim of improving the event selection, the method is robust over a large range of assumed resolutions. A more pessimistic resolution assumption even improves the results.

To study the influence of the resolution assumptions, three smearing configurations $\alpha_i$ are reconstructed with five resolution assumptions $\alpha_j$. The resolution used so far is denoted as $\alpha_2$. The set $\alpha_1$ stands for a resolution value based on stable particle jets [18]. The aim of the configuration $\alpha_3$ is to show that a slight underestimation of the resolution is not harmful to the method. Finally, $\alpha_0$ and $\alpha_\infty$ represent extreme cases. The actual numbers for the different configurations are displayed in Table 4.2.

In Figure 4.8(a) for each combination the percentage of events is shown where the correct permutation was selected. The colour code encodes the convergence rate. All statistical uncertainties are of the order of $10^{-3}$. The decisive variable is the measurement quality of the input represented by the smearing. This is visible in Figure 4.8(a) in the decreasing efficiencies for larger smearing values $\alpha_i$. On the other hand, the assumed resolution mainly influences the convergence rate. If the $\eta_b$ based fit is used (cf. Figure 4.8(b)) the method becomes more sensitive to an underestimation of the resolution.

Although the convergence rate does not strongly depend on the assumed resolution, the quality of the resulting four-vectors does. Their quality is important if the altered
Table 4.2.: Values of the different smearing and resolution assumptions.

<table>
<thead>
<tr>
<th></th>
<th>bottom quarks</th>
<th>light quarks</th>
<th>electrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>10%</td>
<td>5%</td>
<td>2%</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>75%</td>
<td>60%</td>
<td>15%</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>148%</td>
<td>114%</td>
<td>21%</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>210%</td>
<td>180%</td>
<td>29%</td>
</tr>
<tr>
<td>( \alpha_\infty )</td>
<td>1000%</td>
<td>800%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Figure 4.8.: Purities and convergence rates for different combinations of smearing and resolution assumptions. The events were smeared according to the \( \alpha \) set on the ordinate. For the kinematic fitting the resolutions were assumed to be according to the \( \alpha \) set on the abscissa. While the numbers state the percentage of correctly converged events analogue to Figure 4.1.1, the colour code indicates the convergence rate. (a) For the \( p_{\nu,z} \) based fit. (b) For the \( \eta_\nu \) based fit.

Four-vectors are used to improve the resolution of the top quark mass. To quantify the deviation induced by the method, the pull distributions

\[
\frac{(E - E_{\text{Fit}})}{\sigma_{\text{Fit}}}
\]

of the converged correct permutation are fitted with a Gaussian. For the jets and the electron, the width and the mean of the pull distributions are shown in Figure 4.9 for the three smearing configurations \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) in combination with the four resolution hypotheses \( \alpha_1 \), \( \alpha_2 \), \( \alpha_3 \), and \( \alpha_\infty \).

A perfect pull distribution has a width of one and a mean of zero. The method under investigation has a sigma of unity if the resolution and the smearing are equal, but the mean is not centred at zero. This has various reasons: The most severe influence is that the fit is constrained. This is implemented based on the assumption of a Gaussian shape.
Figure 4.9: (a-e) The mean and $\sigma$ of the respective pull distribution. While the blue symbols stand for moderate smearing with $\alpha_1$, the green ones denote smearing with the standard resolution $\alpha_2$, and finally the blue symbols indicate smearing with $\alpha_3$. On the resolution side each symbol type correspond to a specific resolution assumption. (f) shows the legend.
Figure 4.10.: Same as Figure 4.9 but \( \nu \) as fitting variable.
for the top quark and $W^\pm$ boson mass distributions, while in reality they follow a Breit-Wigner distribution\(^4\). Additionally, the unmeasured $\nu$ influences the mean. Treating it via its pseudorapidity stabilises the deviations to some extent as can be seen from Figure 4.10 in comparison to Figure 4.9. Another reason is the relative importance of the individual four-vectors with respect to the $W^\pm$ boson and the top quark within the fitting function. This effect can be seen in the cases with a mismatch in the smearing and resolution assumptions.

4.3. Further Applications of the Method

The fit selection can also be used for b jet identification. Apart from being a selection method, a mass measurement can benefit from the fit results. These applications are discussed in the following sections.

4.3.1. b Jet Identification

As b jet identification is not needed within the selection process and due to the high purity of the selected sample it is possible to use the method for b jet identification. To tag a b jet it is not necessary to select the correct permutation; also the permutation where $b_h$ and $b_\ell$ are exchanged is valid. Furthermore permutations contribute where either $b_h$ or $b_\ell$ is assigned as a b jet, while the other is not. In this case one light jet is tagged incorrectly. Finally, in two permutations both b jets and both light jets are tagged incorrectly.

Without further enhancements the method under investigation tags $(81.6 \pm 0.2)\%$ of the b jets as identified and labels $(17.1 \pm 0.1)\%$ of the light jets wrongly as b jet. The purity can be improved further by a mass range limitation and a $P(\chi^2)$ limit. If both are used as described in Section 4.1.5 a efficiency of $(66.3 \pm 0.2)\%$ is achieved and $(11.1 \pm 0.1)\%$ of the light jets are tagged wrong. All results are given in Table 4.3.

\(^4\)This influence was investigated on a toy Monte Carlo written by Sven Menke, where it is possible to use Gaussian shapes for the mass distributions.

Table 4.3.: b Jet identification performance on parton level for both neutrino implementations (T), a $P(\chi^2)$ limit at 0.15 ($P$), and a mass range limitation between 140 and 200 ($m$). All uncertainties are between 0.1% and 0.2%.

<table>
<thead>
<tr>
<th>T</th>
<th>P</th>
<th>m</th>
<th>Efficiency [%]</th>
<th>Mistag [%]</th>
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<tr>
<td>$p_{\nu,z}$</td>
<td></td>
<td></td>
<td>81.6</td>
<td>17.1</td>
</tr>
<tr>
<td>$\eta_\nu$</td>
<td></td>
<td></td>
<td>81.4</td>
<td>17.6</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td></td>
<td>x</td>
<td>76.9</td>
<td>15.4</td>
</tr>
<tr>
<td>$\eta_\nu$</td>
<td></td>
<td>x</td>
<td>75.8</td>
<td>15.6</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td></td>
<td>x</td>
<td>69.9</td>
<td>12.2</td>
</tr>
<tr>
<td>$\eta_\nu$</td>
<td></td>
<td>x</td>
<td>70.1</td>
<td>12.6</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td></td>
<td>x</td>
<td>66.3</td>
<td>11.1</td>
</tr>
<tr>
<td>$\eta_\nu$</td>
<td></td>
<td>x</td>
<td>65.9</td>
<td>11.4</td>
</tr>
</tbody>
</table>
### 4.3.2. Improvement on the Energy Resolution

As further knowledge is introduced, the resolutions of the different objects as well as of the two $W^\pm$ bosons and the top quark improve.

First, the individual objects are presented. Following the Gaussian assumption on the resolution (Equation 2.1) the normalised deviation

$$\frac{E' - E}{E} \sqrt{E/\text{GeV}}$$

is used. Here $E$ denotes the energies of the generator objects; $E'$ stands for the energy of the objects after smearing, simulating an energy loss, or after altering the four-vectors within the fit procedure. For a fair comparison in all cases only events are considered where the correct permutation is selected by the fit. For the light jets and the b jets, the initial width is seven respectively five percent smaller than the smearing suggests. This confirms that heavily smeared objects, which correspond to badly measured objects, do not converge at all, or do not yield the smallest $\chi^2$ value for the correct combination. The same statement holds for the initial shift of the mean.

The improvement is most prominent for light jets (cf. Figure 4.11(a)), as they are most directly constraint via the $W^\pm$ boson mass constraint. If no energy loss is simulated, the RMS is reduced from $(106 \pm 0.1)$% to $(97.2 \pm 0.1)$%. In contrast, the electron cannot gain much since it has to compete with the neutrino, which has—regardless of the implementation—a much larger uncertainty. Also, the energy loss compensation is largest for the light quarks. Since b jets are only constrained via the more involved top quark mass equality constraint and exhibit larger uncertainties their improvement is much smaller.

If an energy loss is simulated the improvement on the resolution stays unchanged for the light quarks, the loss can even be corrected for by about one fourth. The relative changes for the light jets and the b jets are given in Table 4.4.

Table 4.4.: Improvement of the resolution for the light quarks and the bottom quarks on the smeared only sample, and on the sample with an additional energy loss. The uncertainties are calculated with the pessimistic assumption of being uncorrelated. All numbers are in %.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Smeared</th>
<th>Smeared + Energy Loss</th>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>RMS</td>
</tr>
<tr>
<td>light quarks</td>
<td>$-24.5 \pm 1.0$</td>
<td>$-8.0 \pm 0.1$</td>
</tr>
<tr>
<td>bottom quarks</td>
<td>$-7.9 \pm 6.2$</td>
<td>$-4.2 \pm 0.1$</td>
</tr>
</tbody>
</table>

The improvement in resolution of the individual objects translates into an improvement in resolution for higher level objects like the $W^\pm$ boson mass. Figure 4.12(a) illustrates the course of this chapter. Starting from the MC@NLO simulation (green) the events are smeared to simulate the detector response, which results in the black $m_{W^\pm}$ mass distribution. The mass shown in the black histogram is already based on the jets which are selected by the method. Finally, the stacked histogram shows the result of the fit split up into the different permutation classes. Here, the red histogram denotes the correct permutation and, due to the stacking, the whole distribution after altering the four-vectors within the fit. The orange contribution reflects the two permutations, for
Figure 4.11.: Resolution of energy for (a) the light jets (c) the b jets and (e) the electrons. The black histograms denote the energies which entered the fit. The red histograms are based on the energies after alteration by the fit. In all cases only events were considered, where the correct permutation was selected. (b,d,f) The same but with an additionally simulated energy loss.
which the correct jets are selected but the assignment within the hadronically decaying
top quark is incorrect. Finally, the blue histogram stands for the incorrect permutations.
For comparison, Figure 4.12(b) shows the same with an additional energy loss in the
beginning. Both scenarios exhibit a clear improvement towards the theoretical distribution.
Furthermore, the mass shift caused by the energy loss is partially corrected.
The improvement on the resolution of the $W^\pm$ boson mass is even more prominent
visible in Figure 4.12(c) and Figure 4.12(d), where the relative deviations
\[
\frac{m_{W^\pm,h} - m_{W^\pm,h}^{\text{reco}}}{m_{W^\pm,h}}
\]
of the reconstructed $W^\pm$ boson masses for both scenarios are plotted. While the result for
the dataset with smearing only is symmetric, the sample with an additional energy shift
still possesses a small shift. This shift is mainly present within the fraction of correctly
converged permutations, because a wrong permutation tends to only converge with a
lower $\chi^2$ value than the correct permutation, if it fits better to the assumptions—in this
case the $W^\pm$ boson mass. However it is a considerable improvement.

### 4.3.3. Reconstruction of the Top Quark Mass

As for the $W^\pm$ boson mass the resolution is improved for the top quark mass (Figure 4.13).
The relative deviation of the reconstructed top quark with respect to the simulated value
is drawn in Figure 4.13(c). As can be seen, the top quarks reconstructed from the
combinatorial background tend to have large masses. A fit from $-25\%$ to $25\%$ for the
correct permutations yields a width of $5.2\%$. Compared with the corresponding number
for the initial four-vectors—$7.3\%$—this is an improvement of about $28\%$.

Figure 4.13(a) shows the same quantities as Figure 4.12(a), but for the hadronically
decaying top quark. In contrast to the $W^\pm$ boson mass, a shift is observed in the top quark
mass for the permutations within the correct jet triplet. This shift towards lower values
is caused by the different energy fractions carried by the light and the bottom quarks, as
seen in Section 2.1.1. If a b jet is associated to a light quark, its energy has to be scaled
down on average to fulfill the $W^\pm$ boson mass constraint. On the other hand, the light
jet associated to a bottom quark in these permutations is only altered slightly due to the
less strict top quark mass equality constraint. Thus, on average it cannot compensate the
energy shift within the $W^\pm$ boson system, and therefore a shift remains in the top quark
mass. The split up also shows that all other permutations produce a shallow bump.

Figure 4.13(b) shows the result of the kinematic fit in comparison with the result of
the $p_T^{\text{max}}$ method. The plot contains for each method the result with and without strict
phase space restrictions. The kinematic fit performs better in both cases. While imposing
the strict phase space requirements improves the fitted Gaussian width of the top quark
mass distribution resulting from the $p_T^{\text{max}}$ selection by about $16\%$, it is nearly unchanged
for the mass distribution resulting from the kinematic fit.

To compare the result of the momentum driven implementation with the result of
the pseudorapidity driven approach, both mass distributions are normalised and divided
binwise ($\eta_\nu$ approach in the enumerator). The ratio is shown in Figure 4.13(d). As can
be seen the fit based on $\eta_\nu$ shows a light tendency towards lower top quark masses. This
is because seldomly starts with vanishing $z$ component of the momentum and thus tends
to have a higher energy in the beginning. However, in the interesting region around the
Figure 4.12.: The reconstructed mass distribution of the hadronically decaying $W^\pm$ boson. The green histograms represent the simulated mass distribution. The black histogram shows the distribution for the selected jets after (a,c) smearing and (b,d) after smearing and simulating an energy loss. The hatched histograms are stacked and show the reconstructed mass based on the altered four-vectors split up by permutation class (Table 3.1). (c,d) Show the relative deviation of the reconstructed $W^\pm$ boson mass with respect to the simulated mass, where (c) shows the result for the smeared and (d) the result of the smeared and shifted sample. The colour code is the same as for (a,b).
Figure 4.13.: (a) The reconstructed top quark mass distribution split up by permutation class. The simulated mass is drawn in green (scaled to 10%). The black histogram shows the distribution for the selected jets after smearing. (b) The reconstructed top quark mass for the kinematic fit (red) in comparison with the $p_t^{\max}$ method (blue). Additionally both methods are drawn with the strict phase space requirements imposed. (c) Relative deviation of the reconstructed top quark mass with respect to the simulated mass for the smeared sample. The colour code is the same as for (a). (d) Reconstructed top quark mass of the $\eta_\nu$ implementation normalised binwise to the result of the $p_{\nu,z}$ implementation. Values below 100 GeV are not shown due to lacking statistics. The dotted Line indicates the centre value of the simulated distribution.
centre of the simulated top quark mass (dotted line), both implementations yield the same result.

In summary, in this section it was shown that the selection of the kinematic fit does not bias the top quark mass—neither in the $p_{\nu,z}$ nor in the $\eta_\nu$ implementation. Furthermore, it was shown that if the four-vectors altered within the fitting process are used, the energy resolution of the individual objects, the $W^\pm$ boson and the top quark masses is improved. Finally, the selection can be used for a $b$ jet identification.
5. Method Application at Detector Level

In the following, the method of kinematic fitting with constraints is applied to the full simulation samples. Besides the signal, also selected background channels (cf. Chapter 2) are investigated. Additional means for further suppression of background events are evaluated, and prospects for a mass measurement are explored.

Since for this signal sample the top quark width is not simulated, the top quark mass equality constraint is set strict throughout this chapter. This is only a minor limitation, because, as shown in Section 4.1.3, accommodating the $W^\pm$ boson width is much more important.

5.1. Distribution of Selected Permutations

For events where exactly four jets and exactly one electron are reconstructed the distribution of selected permutations is derived analogue to Section 4.1.1. The resolutions assumed for the fit are those derived in Section 2.1.2 ($\alpha_{u,d,s,c} = 114\%$, $\alpha_b = 148\%$, and $\alpha_\ell = 21\%$). These are also applicable when using the strict phase space requirements. Since the actual permutation can only be identified on the matched subsample (sample 4Jm), there exists a small bias towards better measured and reconstructed objects. The case of non-matched objects will be discussed in Section 5.3.3.

The convergence rate decreases slightly to 98.1\% while it is 98.7\% at parton level. The kinematic fit yields $(60.8\pm0.6)\times0.981\% = (59.6\pm0.6)\%$ selection efficiency, while the $p_t^{\text{max}}$ method yields only $(44.7\pm0.6)\%$ (cf. Figure 5.1). The strict requirements mainly improve

![Figure 5.1: Same as Figure 4.1, but for sample 4Jm.](image)
Discussed next are events where exactly five jets and exactly one electron are reconstructed and all objects can be matched (sample 5Jm). Compared to the first case, one additional jet occurs and thus the number of permutations increases to 60. The higher number of random combinations leads to an increase of the convergence rate to 99.5%. Since the correct permutation is unchanged this gain is only in the wrong permutations. Also, the selection efficiency for the correct jet triplet decreases to $(33.3 \pm 0.6) \times 0.995\% = (33.1 \pm 0.6)\%$. Although this again outperforms the $p_t^{\text{max}}$ method, which discriminates ten permutations in the five jet case and has an efficiency of $(22.4 \pm 0.6)\%$, the selection efficiency is low. Thus, discarding events with five and more reconstructed jets is the better option. Merging jets, which are close in $\Delta R$ and originate in the same vertex would be another alternative. This is motivated by the fact, that the signature of the semileptonic $t \bar{t}$ decay only consists of four partons. Additional partons should emerge from higher order processes if they come from the same vertex.

Imposing the strict phase space requirements increases the selection efficiency to $(35.6 \pm 0.8) \times 0.994\% = (35.4 \pm 0.8)\%$. The results for all permutations are shown in Figure 5.2. Spikes at permutations 48, 49, and 60 indicate cases, in which the additional jet is interchanged with a b jet, while conserving the species (b jet, light jet) of all other objects. In the spike at permutation 19 in addition one light jet is interchanged with $b_h$. These four spikes manifest again that the $W^\pm$ boson mass constraint is dominant.

If only the correct jet triplet shall be selected, the spike at 60 is not harmful as here only $b_\ell$ is affected, and the selected jet triplet remains unchanged. Nevertheless, the mass of the top quark derived from the fit improved four-vectors is changed, due to the top quark mass equality constraint.

Figure 5.2.: Same as in Figure 5.1, but for sample 5Jm and respecting all 60 permutations. The permutations are now indicated by numbers, where one to twelve correspond to the permutations of the four jet case (cf. Table 3.1).
Table 5.1.: Same as in Table 4.1, but for sample 4Jms. All uncertainties are between 0.5% and 0.9%.

<table>
<thead>
<tr>
<th>T</th>
<th>b</th>
<th>P</th>
<th>m_t</th>
<th>Kinematic Fit</th>
<th>p_L^{max}</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
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<td>Correct</td>
</tr>
<tr>
<td>p_{\nu}</td>
<td>97.9</td>
<td>44.4</td>
<td>62.3</td>
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</tr>
<tr>
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<td>x</td>
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</tr>
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</tr>
<tr>
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<td>74.4</td>
<td>83.2</td>
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<td>75.2</td>
<td>84.2</td>
<td>58.2</td>
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<tr>
<td>\eta</td>
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<td>71.3</td>
<td>75.3</td>
<td>83.9</td>
<td>59.8</td>
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Table 5.2.: Same as in Table 5.1, but for sample 5Jms. All uncertainties are between 0.7% and 1.2%.

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<td>100.0</td>
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<td>19.1</td>
<td>35.8</td>
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</tr>
<tr>
<td>\eta</td>
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<td>96.4</td>
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<td>36.5</td>
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<td>28.3</td>
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</tr>
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<td>35.8</td>
</tr>
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<td>28.2</td>
<td>51.2</td>
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<td>p_{\nu}</td>
<td>x x</td>
<td>72.7</td>
<td>54.4</td>
<td>65.9</td>
<td>47.9</td>
</tr>
<tr>
<td>\eta</td>
<td>x x</td>
<td>75.3</td>
<td>54.3</td>
<td>65.4</td>
<td>49.2</td>
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<tr>
<td>p_{\nu}</td>
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<td>54.4</td>
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<tr>
<td>\eta</td>
<td>x x x</td>
<td>69.5</td>
<td>54.8</td>
<td>66.1</td>
<td>46.0</td>
</tr>
</tbody>
</table>
All further enhancements presented in the last chapter—b jet identification, mass range limitation, and $P(\chi^2)$ limit—are applicable. The resulting efficiencies for all configurations are given in Table 5.1 and Table 5.2 for the case with strict phase space requirements and Table D.1 and Table D.2 for the case without strict phase space requirements. As can be seen, in all cases, the $\eta_\nu$ implementation yields slightly better efficiencies for nearly all configurations analysed. The mass range requirement as well as the b jet identification (60 % b jet identification efficiency) are again the most efficient enhancements. Using both, the purity can be increased to (63.6 ± 1.0)% for the five jet events, which allows the usage of these events as well.

In summary, it was shown in this subsection that the kinematic fit achieves the highest performance in the $\eta_\nu$ implementation and on four jet events. Also, the jet triplet selected by the kinematic fit has a higher purity than the one selected with the $p_T^{\text{max}}$ method.

### 5.1.1. Rejection of Background

Due to the fact, that in data beside signal events also background events are present the possibilities to reject them via kinematic fitting are investigated in the following. As stated in Section 2.2.2 for the background events strict phase space requirements are needed. Therefore, they are applied throughout this chapter.

Since background events do not fulfil the assumptions of the kinematic fit, it converges less likely or results in lower $\chi^2$ probabilities than the signal. In contrast, a rejection via the use of the $p_T^{\text{max}}$ method is not possible, as a triplet is selected for each event and the method does not offer a quality measure.

Table 5.3 gives the fraction of selected events for the $t\bar{t}$ signal and the $W + n$ jets samples. The numbers for the QCD background events are not given due to lacking statistics. The signal sample is split up into the $e +$ jets channel and all other decay channels despite the all-hadronic one ($t\bar{t} \rightarrow \text{rest}$). Due to the fact that there was neither performed a b jet identification for the available Monte Carlo samples nor the Monte Carlo information is available, only the numbers for $P(\chi^2)$ and mass range limits are given.

For the $e +$ jets channel, the convergence rate drops to 86.8%, since the sample also
contains events, where a matching is not possible. These events tend to be less well measured. As an example at least one object is not reconstructed or measured accurately enough to be identified with an object of the hard interaction. On the background side, for four jet events the convergence rate is between 69.7% and 77.0%. The increase in random combinations for five jet events affects particularly the convergence rates of background events. If five jet events shall be included at least a $P(\chi^2)$ limit should be set. Even better would be the use of b jet identification, since b jets are rare in these events.

While in the $e + \text{jets}$ channel the applied $P(\chi^2)$ limit inhibits additional 24.0% of the events from being selected, the background channels are reduced in average by about 30% absolute.

As can be seen, the $\eta_{\nu}$ implementation performs in the majority of configurations slightly better than the $p_{\nu,z}$ implementation; the signal convergence rates are higher and the ones of the backgrounds are lower.

Due to its better overall performance and in combination with the flatter $P(\chi^2)$ distribution\(^1\) the $\eta_{\nu}$ implementation of the fit is used as standard in the following instead of the one based on $p_{\nu,z}$.

\(^1\)The statement made in Section 4.2.1 was verified to be valid for the full simulation sample as well.
5.2. $P(\chi^2)$ Distribution

In the following the characteristics of the $P(\chi^2)$ distribution are discussed. The $P(\chi^2)$ distribution of the signal sample shows the same characteristics as the parton level distribution. Figure 5.3 compares the $P(\chi^2)$ distributions of four signal subsamples for the $\eta\nu$ implementation of the fit. Due to the higher count of permutations in the five jet scenarios, which can result in small $\chi^2$ values, their probability distributions are biased more towards one. On the other hand, the unmatched samples (sample 4Jas & 5Jas) exhibit a pronounced peak at low probabilities. This reflects the fact that in about 70% of the events not all objects can be matched.

In Figure 5.4 the $P(\chi^2)$ distribution is plotted against the reconstructed mass of the hadronically decaying top quark, calculated from the altered four-vectors. Sample 4Jms is used, to allow a separation of correct permutations (a), the correct triplet (b), and incorrect permutations (c). In the first two scenarios the mass distribution is narrow. In contrast, it is broad for combinatorial background. This explains the higher purity when using a mass range requirement.

On the other hand, a requirement on $P(\chi^2)$ helps to reduce background events as can be seen in Figures 5.4(d)-(f), where the corresponding distributions are shown for the other $t\bar{t}$ decay channels and the $W + n$ jets background sample. For these samples, the distributions are much broader, as they contain events that are intrinsic in contradiction to the assumptions of the kinematic fit. Therefore, a limit on $P(\chi^2)$ has a larger impact on these samples. Due to the lack of statistics and the large uncertainties the QCD samples cannot be discussed under this aspect.

![Figure 5.3: $P(\chi^2)$ distribution of the selected permutations for four different samples (4Jas, 5Jas, 4Jms, and 5Jms). All histograms are based on the $\eta\nu$ implementation and with strict phase space requirements.](image-url)
5.2 $P(\chi^2)$ Distribution

Figure 5.4: The $P(\chi^2)$ probabilities versus the reconstructed mass of the hadronically decaying top quark, $m_{t,h}$. (a-c) For e+jets channel, (d) for the non e+jets decays within the signal sample ($t\bar{t} \rightarrow \text{rest}$), and (e,f) for the used W+n jets channels.
5.3. Further Applications of the Method

As for the parton level an outlook for further applications is given in the following sections. In Section 5.3.1 the possibilities for b jet identification are reviewed. The improvement on the resolution of the individual objects as well as on the $W^{\pm}$ boson mass are discussed in Section 5.3.2. Finally, the shape of the reconstructed top quark mass distribution is investigated in Section 5.3.3.

5.3.1. b Jet Identification

Analogue to Section 4.3.1 a rough estimate for the b jet identification power of the method will be given. Additionally, the influence of falsely tagged background will be discussed.

Without further enhancements the method under investigation tags $(76.7 \pm 2.1)\%$ of the b jets as identified and labels $(22.3 \pm 1.0)\%$ of the light jets wrongly as b jet. The purity can be improved further by a mass range limitation and a $P(\chi^2)$ limit. If both are used as described in Section 4.1.5 the efficiency is reduced to $(59.0 \pm 1.9)\%$. However only $(13.8 \pm 0.8)\%$ of the light jets are tagged wrongly. All results are given in Table 5.4.

Since in background b jets are uncommon, almost every selected background event results in a misidentified light jet. Thus strict requirements should be imposed on the quality of the fit as well as on the reconstructed top quark mass.

5.3.2. Improvement of the Resolution

Using sample 4+Jms, the improvement of the energy resolution of individual objects is determined as in Section 4.3.2. Due to the pronounced tail of the bottom quark distribution and the different strength of the distinct constraints, only the resolution for the light jets changes significantly. This results in an relative improvement of $(4.9 \pm 0.7)\%$. The change in the mean is, in contrast to the parton level sample, not significant for light jets.

Comparing the widths and means of fitted Gaussians for the distributions shown in Figure 5.5 to the respective ones of all matched events, it becomes visible that outliers tend to not converge to the correct permutation. This is the same effect as at parton level.

Table 5.4.: Same as Table 4.3, but for sample 4Jm.

<table>
<thead>
<tr>
<th>T</th>
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<th>m</th>
<th>Efficiency</th>
<th>Mistag</th>
</tr>
</thead>
<tbody>
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<td>$p_{\nu,z}$</td>
<td></td>
<td></td>
<td>75.3 ± 2.0</td>
<td>22.6 ± 1.1</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td></td>
<td>76.7 ± 2.1</td>
<td>22.3 ± 1.0</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>x</td>
<td>68.7 ± 2.0</td>
<td>19.3 ± 1.0</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>x</td>
<td>70.6 ± 2.0</td>
<td>19.5 ± 1.0</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>x</td>
<td>61.2 ± 1.9</td>
<td>15.6 ± 0.9</td>
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<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>x</td>
<td>63.1 ± 1.9</td>
<td>15.4 ± 0.9</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>x</td>
<td>56.9 ± 1.8</td>
<td>13.6 ± 0.8</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>x</td>
<td>59.0 ± 1.9</td>
<td>13.8 ± 0.8</td>
</tr>
</tbody>
</table>
5.3 Further Applications of the Method

Figure 5.5: Energy resolution for (a) the light quarks, (b) the bottom quarks, and (c) the electrons. The black histograms denote the energies which entered the kinematic fit. The red histograms are based on the energies after alteration by the fit. In all cases only events were considered, where the correct permutation had the smallest $\chi^2$.
Figure 5.6.: Relative deviation of the reconstructed $W^\pm$ boson mass with respect to the simulated mass. In (a) the initial four-vectors are used and in (b) the fit improved ones.

In Figure 5.6 the relative deviation of the reconstructed $W^\pm$ boson mass is shown using the initial four-vectors (a) and the fit improved four-vectors (b). The width of the distribution for the correct permutations improves from $(7.8 \pm 0.2)\%$ to $(4.2 \pm 0.0)\%$, where $\pm 0.0$ stands for an uncertainty of less than 0.1. As can be seen the change occurs with almost same strength in all permutation classes and is highly asymmetric. This could be a possible source of bias and has to be kept in mind when using the altered four-vectors.

5.3.3. Reconstruction of the Top Quark Mass

In the following, the prospects for a top quark mass measurement using the selection of the kinematic fit are discussed. First the systematic differences in the resulting top quark mass distributions are investigated for the kinematic fitting method and for the $p_t^{\text{max}}$ method. Closing an outlook for first year LHC data is given.

**Luminosity Rescaling**

For a comparison between Monte Carlo simulation and data of a specific luminosity $\mathcal{L}_{\text{data}}$ the different Monte Carlo samples need to be scaled.

To assure that the fluctuations of the modified Monte Carlo sample reflect the fluctuations in a data sample of the same luminosity the following approach is chosen for the scaling process: For each sample the number of expected events $N_{\text{ex}}$ is calculated with the formula

$$N_{\text{ex}} = N_{\text{sel}} \times f,$$
where $N_{\text{sel}}$ denotes the number of selected events. The scaling factor $f$ is defined as:

$$f := \mathcal{L}_{\text{data}} \frac{\sigma}{N}.$$ 

Here, $\sigma$ is the theoretical cross section of the respective decay process and $N$ is the weighted number of Monte Carlo events available. As the background samples are simulated at leading order while the signal sample is simulated at next-to-leading order, the cross sections of the background samples are renormalised with K-factors$^2$. As a last step, a random number is drawn from a Gaussian distribution with width $\sqrt{N_{\text{ex}}}$ and mean $N_{\text{ex}}$. This corresponds to the actual number of events chosen for a certain sample.

To avoid an underestimation of statistical uncertainties upscaling should be avoided. Unfortunately, this rule can not be obeyed for the QCD samples due to a lack of statistics; it rather has to be scaled up by huge factors. In combination with the fact that there are only a handful of events passing the whole selection process, this would produce meaningless spikes in all distributions. Thus, the shape of the $W + n$ jets background events are used as an approximation for the shape of the QCD background and only the normalisation is taken from the central values predicted by the QCD simulation.

The lowest simulated luminosity available for the other samples is the one of the $W + 5$ jets background. This is used for the investigation on the systematic differences of the methods. For the outlook on first years data, 200 $\text{pb}^{-1}$ is chosen. The scaling factors $f$ for the different samples are given in Table 5.5.

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<td>$W + 5$ jets</td>
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<td>0.25</td>
</tr>
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<td>20.20</td>
</tr>
<tr>
<td>J4N4</td>
<td>85.11</td>
<td>21.28</td>
</tr>
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<td>45.45</td>
</tr>
<tr>
<td>J4N5</td>
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<td>J5N5</td>
<td>2.74</td>
<td>0.68</td>
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</table>

### Top Quark Mass

In order to measure the top quark mass, its distribution is reconstructed from the four-vectors of its decay particles as the invariant mass of the selected triplet. This is the same procedure as described for the parton level sample (cf. Section 4.3.3). In all figures of this section a solid black line indicates the generated top quark mass and a dotted black line stands for the mean of the top quark mass distribution reconstructed from the initially reconstructed four-vectors of the correct triplet (cf. Section 2.1.2).

A separation into several permutation classes (correct, triplet correct, wrong), reveals the same bias as seen at the parton level: a shift towards lower masses for the permutations

$^2$K-factors are defined as the ratio of NLO and LO cross-sections. They are used to rescale the normalisation of the LO predictions, while preserving the LO four-vector kinematics.
within the correct triplet (cf. Figure 5.7). Nevertheless, the peak position of the top quark mass distribution is closer to the simulated value, although the change of the jet energy scale is small for the individual objects. There is little combinatorial background (blue) and its shape exhibits only a shallow bump in the region of the simulated top quark mass. The cut-off of the distribution at 100 GeV is a result of the $W^\pm$ boson mass constraint in combination with the fact that all objects are required to have a transverse momentum of more than 20 GeV.

To explore what can be expected from an analysis of LHC data, matching is not used in the following. In Figure 5.8 all selected events with four reconstructed jets (sample 4Jas) are used to reconstruct the top quark mass. Since the additional non-matched events are measured and/or reconstructed less accurate, their resolution is worse and results in broader distributions. If the jet triplet is selected with the $p_t^{\text{max}}$ method (black), the result is a broad distribution with pronounced shoulders. The kinematic fit is an improvement compared to the $p_t^{\text{max}}$ method even without using the altered four-vectors (blue). If these are used (red), the result improves even more, being visible in a considerably reduced width of the mass distribution peak. There is also a slight improvement of the mean towards the simulated top quark mass.

To compare the shapes of the reconstructed top quark mass distributions, the histograms of Figures 5.7 and 5.8 are normalised to unit area, and divided by the result of the kinematic fit using the unmatched sample 4Jas (red in Figure 5.8). In addition, the same is done for the kinematic fit with a $P(\chi^2)$ limit at 0.15. The result is shown in Figure 5.9.
Figure 5.8.: The invariant mass of the selected jet triplet for the hadronically decaying top quark (red). For comparison the invariant mass of the same jet triplet, but using the initial four-vectors (blue) is drawn. In addition, the invariant mass resulting from the $p_t^{\text{max}}$-selection is drawn in black. These histograms are based on sample 4Jas. The solid line indicates the simulated mass, the dashed one the mass resulting from the initially reconstructed four vectors of the correct triplet (cf. Section 2.1.2).

The broader distribution of the unmatched sample 4Jas with respect to the matched sample 4Jms is reflected in this plot in the bump of the red histogram in the region between 150 GeV and 180 GeV. Applying a $P(\chi^2)$ limit (green) reduces the contribution of the tails considerably and thus reduces the deviation from the pure fit in the peak region to 10%. The most prominent feature in the distribution based on the $p_t^{\text{max}}$ method (black) is a peak at low masses. This reflects the lack of the $W^{\pm}$ boson mass constraint induced cut-off in the $p_t^{\text{max}}$ distribution. This method also has the lowest fraction of entries within the peak region. The blue data points show the effect of the kinematic fit on the shape by altering the four-vectors. As an example, there are about one fifth less entries in the peak region of the distributions based on the initial four-vectors than with the fit improved ones. Thus, in the following the fit improved four-vectors are used.

Next, the effects of the inclusion of background are investigated. As outlined above, these systematic analyses are carried out on a luminosity of 800 pb$^{-1}$. For Figure 5.10(a) only the four jet events of the $t\bar{t}$ and the background samples are used. The $e +$ jets channel shows a clear peak. In contrast, the distribution of the other decay channels of the $t\bar{t}$ pairs is flat as is the distribution of the $W + n$ jets background and especially does not peak in the region of the top quark mass. This is another indication, that the method does not bias the selection. The contribution of the QCD background events is
Figure 5.9: Binwise ratio of the normalised histograms shown in Figures 5.7 and 5.8 and the histogram based on sample 4Jas (red in Figure 5.7). Additional, the same is shown for a kinematic fit with a $P(\chi^2)$ limit at 0.15 (green). All other colour codes are identical to Figures 5.7 and 5.8. The solid line indicates the simulated mass, the dashed one the mass resulting from the initially reconstructed four vectors of the correct triplet (cf. Section 2.1.2).

rather small, but for reliable statements it must be derived from data.

In Figure 5.10(b), the result of the kinematic fitting method is compared to the result of the $p_{t}^{\text{max}}$ method. To get an estimate of the peak position, a ±20 GeV mass range window around the maximum is fitted with a Gaussian. Both methods show a shift from the mass resulting from uncorrected four-vectors, (160.7 ± 0.2) GeV, to the simulated mass of 172.5 GeV. The mass resulting from the peak of the kinematic fit result distribution is (167.2 ± 0.4) GeV. For the distribution of the $p_{t}^{\text{max}}$ method the peak value is at (164.2 ± 0.8) GeV. So, the correction is a bit larger for the kinematic fitting method and the peak comes closer to the simulated mass. Also, the resulting peak is much narrower for the kinematic fitting method.

Figures 5.10(c)-(d) show the same as Figures 5.10(a)-(b), but with the requirement $P(\chi^2) > 0.15$ in addition. Besides reducing the overall background level, this requirement reduces especially the contribution of combinatorial background in the right shoulder. The $p_{t}^{\text{max}}$ method based distribution remains unchanged. For the kinematic fit the peak position—in this case at (166.0 ± 0.4) GeV—is nearly unchanged. Nevertheless, a more sophisticated mass determination can profit from the cleaner environment. This is reflected in the signal to background ratio $S/B$ shown in Figure 5.11(a). Without a $P(\chi^2)$ limit, the kinematic fit has an average $S/B$ ratio of 0.9±0.1 and is nearly indistinguishable from the $p_{t}^{\text{max}}$ method, which has 0.8 ± 0.1. Split up into mass regions, the result is
Figure 5.10.: The reconstructed top quark mass for a luminosity of 800 pb$^{-1}$ for signal and background. (a,c) The contribution of the different channels for the kinematic fitting method. (b,d) The result of the kinematic fit in comparison with the $p_t^\text{max}$ method. The solid line indicates the simulated mass, the dashed one the mass resulting from the initially reconstructed four vectors of the correct triplet (cf. Section 2.1.2).
Figure 5.11.: The binwise and average Signal/Background Ratio for the different methods. The solid line indicates the simulated mass, the dashed one the mass resulting from the initially reconstructed four vectors of the correct triplet (cf. Section 2.1.2).

more clearly in favour of the kinematic fit. In the peak region bin of the respective distributions, S/B is $2.6 \pm 0.2$ vs. $1.7 \pm 0.1$ for the kinematic fit and the $p_{t}^{\text{max}}$ method, respectively. Imposing the $P(\chi^2)$ limit improves this to $4.9 \pm 0.5$ for the kinematic fit in the maximum region and to $1.3 \pm 0.2$ on average.

The result for the five jet events is shown in Figure 5.12. Due to the lower discrimination power against combinatorial background (cf. Table 5.2), the mass distribution is considerably broader than for the four jet events. Nonetheless, it is again much narrower
than the $p_t^{\text{max}}$ distribution. The Gaussian fit of the peak has a mean of $(164.0 \pm 0.5)$ GeV for the distribution of the kinematic fit and of $(162.5 \pm 0.8)$ GeV for the $p_t^{\text{max}}$ method. However, as can be seen from the broad distribution resulting from the $p_t^{\text{max}}$ method, the kinematic fit is the better choice if the five jet events should be included. The average signal to background ratio (cf. Figure 5.11(b)) is $1.7 \pm 0.2$ for the kinematic fit with $P(\chi^2)$ limit. It is higher than for the four jet events, since the background events are suppressed by an additional factor $\alpha_s$. However in the peak region it is only $3.3 \pm 0.3$.

By combining all four and five jet events of the signal and the various background
Table 5.6.: Deviation of the peak position from the simulated mass of 172.5 GeV and from the mass initially reconstructed from the initial four-vectors of the correct triplet of (160.7 ± 0.2) GeV. The $P(\chi^2)$ limit is always applied.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Kinematic fit [GeV]</th>
<th>$p_t^{\text{max}}$ method [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>172.5 GeV</td>
<td>160.7 GeV</td>
<td>172.5 GeV</td>
</tr>
<tr>
<td>4 jets</td>
<td>−6.5 ± 0.4</td>
<td>5.3 ± 0.4</td>
</tr>
<tr>
<td>5 jets</td>
<td>−8.5 ± 0.5</td>
<td>3.3 ± 0.5</td>
</tr>
<tr>
<td>4 &amp; 5 jets</td>
<td>−7.3 ± 0.4</td>
<td>4.5 ± 0.4</td>
</tr>
</tbody>
</table>

samples (Figure 5.13), a top quark mass of (165.2 ± 0.4) GeV using the kinematic fit and of (163.3 ± 0.6) GeV using the $p_t^{\text{max}}$ method is obtained. Since this is lower than the mass reconstructed of the four jet events and the measurement will be limited by systematics anyhow, the five jet events should not be used.

This section demonstrated how selected events translate into top quark mass distributions. The results for the mass peaks are summarised in Table 5.6. As can be seen, the jets selected and altered by the kinematic fitting method obtain top quark masses closer to the simulated value. Another advantage over the $p_t^{\text{max}}$ method are the narrower peaks of the mass distributions. The cleanest distribution is obtained by using exclusively the four jet events together with a $P(\chi^2)$ limit.

5.3.4. Outlook on First Years Data

Finally, an outlook on the data that is planned to be collected in the first year of LHC is given. A luminosity of 200 pb$^{-1}$ is assumed, and only the four jet events are taken into account. The $P(\chi^2)$ limit is applied to the kinematic fitting method. The resulting top quark mass distribution for a generated mass of 172.5 GeV is shown in Figure 5.14. The fitted peak position is (167.8 ± 1.2) GeV, and the corresponding value for the $p_t^{\text{max}}$ method is (163.2 ± 1.1) GeV.

![Figure 5.14.: Reconstructed top quark mass for a luminosity of 200 pb$^{-1}$.](image-url)
6. Conclusions and Outlook

In this thesis, implementations, applications, and prospects of a kinematical fit with constraints for the $t\bar{t} e+\text{jets}$ channel were presented. The fit was based on the assumptions that the directions of the jets are well measured, while the energies have larger uncertainties and can be improved by a recalibration. The measured $W^\pm$ boson mass, and the equality of the top quark and anti top quark masses were imposed as constraints formulated via lagrangian multipliers. To cope with the intrinsic widths of the $W^\pm$ boson mass and the top quark mass difference, these were realised as loose constraints. This approach exhibited superior results than strict constraints [55].

Two different treatments of the $\nu$ were investigated. The current standard approach is based on the calculation of the $\nu$ longitudinal momentum via the $W^\pm$ boson mass constraint. An alternative approach makes use of the pseudorapidity of the $\nu$. It could be shown, that the alternative approach is more stable and performs better in the discrimination of signal and background events.

It could be shown, that the kinematic fitting method used for signal events improves the selection of the events used for a mass determination and is able to select the correct jet triplet within an event with high efficiency. On four jet events, the correct triplet is selected for $(64.0 \pm 0.7)\%$ of the events, while the current standard method, the $p_t^{\text{max}}$ method, only selects $(54.2 \pm 0.7)\%$. Additional requirements, like $P(\chi^2)$ limits or top quark mass windows for the kinematic fit method can further increase the purity to about $78\%$. In addition, the fit improves the resolution of the top quark mass by altering the four vectors and corrects energy losses induced by the measurement process by approximately one percent.

Due to nonconverging fits, the kinematic fitting method rejects approximately additional $25\%$ of the background events, compared to the $p_t^{\text{max}}$ method. Further requirements can increase the rejection power to $88.7\%$.

The selection can also be used for b jet identification. When using this, $(76.7 \pm 2.1)\%$ of the b jets are tagged correctly while $(22.3 \pm 1.0)\%$ of the light jets are misidentified. The fraction of correctly identified b jets is comparable to other approaches based on constrained fitting [55]. The misidentification rate was not studied there.

For the future, a clever treatment of events with more than five reconstructed jets would be desirable. Furthermore, an implementation based on a maximum likelihood function is promising. This would allow for a more precise treatment of the Breit-Wigner shape of the mass distributions of the $W^\pm$ boson and the top quark.

In conclusion, the implementation of the kinematic fit presented in this thesis can be used on first data not only as a selection tool but also to recalibrate the four vectors used to reconstruct the top quark mass. It could be shown that it outperforms the currently used $p_t^{\text{max}}$ method.
A. Reconstruction of the z-Component of the Neutrino Momentum

The z-component of the neutrino momentum can be derived from the leptonic $W^\pm$ boson mass constraint.

$$0 = (p_\ell + p_\nu)^2 - m_{W}^2 = p_\ell^2 + 2(E_\ell E_\nu - \vec{p}_\ell \vec{p}_\nu) - p_\nu^2 - m_{W}^2$$

The masses of the leptons are neglected and $E_\nu = |\vec{p}_\nu| = \sqrt{p_{\nu,x}^2 + p_{\nu,y}^2 + p_{\nu,z}^2}$. This yields

$$p_{\nu,x}^2 + p_{\nu,y}^2 + p_{\nu,z}^2 = \frac{1}{E_\ell^2} \left( \frac{m_W^2}{2} + p_{\ell,x} p_{\nu,x} + p_{\ell,y} p_{\nu,y} + p_{\ell,z} p_{\nu,z} \right)^2$$ \hspace{1cm} (A.1)\[= \frac{1}{E_\ell^2} \left( a^2 + 2a p_{\ell,z} p_{\nu,z} + p_{\ell,z}^2 p_{\nu,z}^2 \right) , \hspace{1cm} (A.2)$$

with $E_\ell^2 - p_{\ell,z}^2 \approx p_{\ell,t}^2 - p_{\ell,z}^2 = p_{\ell,t}^2$, the transverse momentum of $\ell$. Solving this for $p_{\nu,z}$

$$p_{\nu,z}^{(1,2)} = \frac{2a p_{\ell,z} \pm \sqrt{4a^2 p_{\ell,z}^2 - 4p_{\ell,t}^2 (p_{\ell,z}^2 + p_{\nu,y}^2) E_\ell^2 - a^2}}{2p_{\ell,t}^2}$$

$$\approx \frac{a}{p_{\ell,t}^2} p_{\ell,z} \pm \frac{E_\ell}{p_{\ell,t}^4} \sqrt{a^2 - p_{\ell,t}^2 (p_{\ell,z}^2 + p_{\nu,y}^2)}$$ \hspace{1cm} (A.3)\hspace{1cm} (A.4)$$

is obtained.
B. MC@NLO Steering file

The steering file for MC@NLO. The variable IL1CODE and IL2CODE were switched after 200 000 events to avoid spin effects (cf Section 2.43 of [23]).

# This is a bash script that compiles and runs all of the MC@NLO codes.
# On your system, you need:
# # bash shell AND gmake
# # which are rather standard (ask your system manager if they are not
# # available).
# # HOW TO USE THIS SCRIPT:
# Look for "physical parameters" and "other input parameters" in
# in this file; they control all the inputs for the MC@NLO codes.
# After having modified them to suit your needs, execute this
# file from a bash shell. Notice that the only command in this
# file is
# runMCatNLO
# which is what you need in order to obtain MC@NLO results. Other
# commands are available: see at the bottom of this file for a
# list of them. In this version, they are all commented out;
# uncomment them if you need them.
# # WHAT THE USER MUST DO PRIOR TO RUNNING
# The files
# mcatnlo_hwdriver.f mcatnlo_hwlhin.f
# must be edited in order to insert the 'INCLUDE HERWIGXX.INC' command
# relevant to the version of HERWIG your are going to use. The file(s)
# mcatnlo_hwanXXX.f
# contain sample analysis routines, and must be edited for the same reason.
# Notice, however, that these analysis routines are provided here to furnish
# a ready-to-run package, but they are identical to standard HERWIG analysis
# routines, and should therefore be replaced with your analysis routines.
# In this case, you will simply set the variable HWUTI (in this file) equal
# to the list of object files you need in your routines.
# Finally, the variable HERWIGVER below must be set equal to the name
# of your preferred version of HERWIG (matching the one whose common
# blocks are included in the files above)

# #!/bin/bash

#
# physical parameters
#
#
# CM energy
ECM=14000
# renormalization scale factor
FREN=1
# factorization scale factor
FFACT=1
# mass of the heavy quark (bottom for IPROC=-1705, top otherwise, including
# Higgs production)
HVQMASS=170.9
# width of the top. A negative entry will force the code to compute the width
# at the LO in the SM, in ttbar and single top production and when the
# tops decay
TWIDTH=1.4
# W mass
WMASS=80.398
# W width. A negative entry will force the code to compute the width
# at the LO in the SM, in single top production (Wt channel) when the
# top and W decay, and in WW production when the W's decay
WWIDTH=2.140
# Z mass
ZMASS=91.1875
# Z width
ZWIDTH=2.4952
# branching ratio for Sum_j (top -> l nu_l b_j), with b_j any down-type
# quark, and l a given lepton species. Lepton universality is assumed
BRTOPTOLEP=0.1111
# branching ratio for Sum_ij (top -> u d_i b_j), with d_i and b_j any
# down-type quarks. Flavour universality is assumed
BRTOPTOHAD=0.3333
# branching ratio for W -> l nu_l, with l a given lepton species.
# Lepton universality is assumed
BRWTOLEP=0.1111
# branching ratio for Sum_i (W -> u d_i), with d_i any
# down-type quarks. Flavour universality is assumed
BRWTOHAD=0.3333
# Higgs mass
HGGMASS=120
# Higgs width: MC@NLO does not compute the SM width associated with the
# mass set in HGGMASS. The user must set the width by hand
HGGWIDTH=0.0049
# In the computation of the Higgs cross section at the Born level:
# IBORNHGG=1 --> exact M_top dependence, IBORNHGG=2 --> M_top -> infinity
IBORNHGG=1
# When the mass of a particle P is distributed according to Breit-Wigner
# (which happens in production for the Drell Yan process if P is a W, Z,
# or photon, and in decay if P is a top, a vector boson, or a Higgs),
# the mass range is (if PGAMMAX>0)
# MO_P - PGAMMAX * WIDTH < M_P < MO_P + PGAMMAX * WIDTH
# with MO_P the pole mass of P, and WIDTH its width. If PGAMMAX<0 then
# PMASSINF < M_P < PMASSSUP
# Valid shell variables correspond to
# \( P = V_1, V_2, T_1, T_2, H \)
# for vector boson, top, and Higgs respectively. In the case of top decay,
# the shell variables with prefix \( V_j \) are relevant to the \( W \)'s emerging from
# the decay of the top whose shell variables have prefix \( T_j \).
# When there is only one vector boson or one top in the final state,
# the relevant shell variables have prefix \( V_1 \) or \( T_1 \). In the case of
# vector boson pair production, the prefixes \( (V_1,V_2) \) correspond to \( (W^+,W^-) \),
# \( (Z,Z) \), \( (W^+,Z) \), and \( (W^-,Z) \) for \( \text{IPROC}=-2850, -2860, -2870, \) and 2880
# respectively. In the case of \( t\bar{t} \) production, \( (T_1,T_2) \) correspond
# to \( (t,t\bar{t}) \), and \( (V_1,V_2) \) to \( (W^+,W^-) \) emerging from \( (t,t\bar{t}) \) decays.
# In the case of \( tW^- \) production, \( T_1 \) and \( V_2 \) correspond to \( t \) and \( W^- \) produced
# in the hard reaction respectively (in version 3.4, off-shell effects are
# however not implemented yet), and \( V_1 \) to the \( W^+ \) emerging from the \( t \) decay.

\( V1\text{GAMMAX}=30 \)
\( V1\text{MASSINF}=0 \)
\( V1\text{MASSSUP}=0 \)
\( V2\text{GAMMAX}=30 \)
\( V2\text{MASSINF}=0 \)
\( V2\text{MASSSUP}=0 \)
\( T1\text{GAMMAX}=30 \)
\( T1\text{MASSINF}=0 \)
\( T1\text{MASSSUP}=0 \)
\( T2\text{GAMMAX}=30 \)
\( T2\text{MASSINF}=0 \)
\( T2\text{MASSSUP}=0 \)
\( H\text{GAMMAX}=30 \)
\( H\text{MASSINF}=0 \)
\( H\text{MASSSUP}=0 \)

# quark and gluon masses (used only by HERWIG)
\( U\text{MASS}=0.32 \)
\( D\text{MASS}=0.32 \)
\( S\text{MASS}=0.5 \)
\( C\text{MASS}=1.55 \)
\( B\text{MASS}=4.95 \)
\( G\text{MASS}=0.75 \)

# absolute values of the CKM matrix elements; used for single-top production
# and subsequent top decay, and for top decay in \( t\bar{t} \) production.
# Set \( VUD=VUS=VUB=0 \) to use the defaults in the code
\( VU\text{D}=0.9748 \)
\( VU\text{S}=0.2225 \)
\( VU\text{B}=0.0036 \)
\( VC\text{D}=0.2225 \)
\( VC\text{S}=0.9740 \)
\( VCB=0.041 \)
\( VTD=0.009 \)
\( VTS=0.0405 \)
\( VTB=0.9992 \)

# Set \( \text{AEMRUN=YES} \) to use running \( \alpha_{em} \), \( \text{AEMRUN=NO} \) to use the Thomson value
# process number; MC@NLO process codes are negative. A positive process
# code may be used (executing runMC) to run standard HERWIG
\( \text{IPROC}=-11706 \)
# vector boson code: IVCODE=-1,0,1 for W^-, Z, and W^+ respectively.
# This variables is only used in WH and ZH production
IVCODE=1
# IIL1CODE determines the identities of decay products of tops or
# vector bosons, when spin correlations are included.
# Set IIL1CODE=7 for undecayed vector bosons or tops.
# IIL1CODE is relevant to WH, ZH, single-top, ttbar, and vector boson
# pair production; in the latter two cases, and in Wt production, the
# variable IIL2CODE is also needed. See the manual for a list of valid
# values for IIL1CODE and IIL2CODE. In the case of VV, ttbar and tW
# production, (IIL1CODE,IIL2CODE) control the decays of (t,tbar), (t,W),
# (W^+,W^-), (Z,Z), (W^+,Z), and (W^-,Z) for IPROC=-1706, -2030, -2850,
# -2860, -2870, and 2880 respectively
IIL1CODE=1
IIL2CODE=5
# type of top decay: set TOPDECAY=Wb to allow only t->Wb decays; set
# TOPDECAY=ALL to allow all t->W+down-type-quark decays. In the latter
# case, the flavour of the down quark is determined using the CKM
# matrix elements entered here
TOPDECAY=Wb
# set WTTYPE=REMOVAL to perform the computation of the Wt cross section in
# the Diagram Removal (DR) scheme. Set WTTYPE=SUBTRACTION to use the
# Diagram Subtraction (DS) scheme. See JHEP 0807:029,2008 [arXiv:0805.3067]
WTTYPE=REMOVAL
# ptveto value, used for factorization scale computation if FFACT<0, and
# for renormalization scale computation if FREN<0. Effective only for Wt
PTVETO=50
# incoming left beam
PART1=P
# incoming right beam
PART2=P
# PDF group name; unused when linked to LHAPDF
PDFGROUP=CTEQ
# PDF set number; use LHAGLUE conventions when linked to LHAPDF
PDFSET=10350
# Lambda_5, used in NLO computations. A negative entry returns the value
# resulting from PDF fit.
# WARNING: negative entries may lead to inconsistent results when using
# PDFLIB or LHAPDF: use a positive entry when in doubt
LAMBDAFIVE=0.227
# Scheme
SCHEMEOFPDF=MS
# Lambda_5, used by HERWIG. A negative entry returns the HERWIG default value
LAMBDAHERWIG=-1
# other input parameters
#
# prefix for Bases files; relevant to the integration step
FPREFIX=ttbar
# prefix for event file; relevant to the event generation step
EVPREFIX=ttbar
# prefix for the NLO and MC executables
EXEPREFIX=ttbar
# number of events; set it to 0 to skip the event generation step
NEVENTS=200000
# 0 for weights=+1/-1, 1 for weights whose sum is the total rate
WGTTYPE=1
# seed for random numbers in the generation of events. 0 is default
RNDEVSEED=0
# set BASES=ON to perform integration, =OFF to skip the integration step
BASES=ON
# set PDFLIBRARY=THISLIB, =PDFLIB, or =LHAPDF to obtain PDFs from our
# private PDF library, from PDFLIB or from LHAPDF respectively
PDFLIBRARY=LHAPDF
# set HERPDF=DEFAULT to use HERWIG default PDFs, HERPDF=EXTPDF to use
# the same PDFs as used in the NLO; the setting of this parameter is
# independent of the setting of PDFLIBRARY
HERPDF=DEFAULT
# the variable HWPATH must be set equal to the name of directory
# which contains the version of HERWIG the user wants to link
# to his code
HWPATH="/home/pcl225/nisius/MCatNLO/HERWIG6510/"
# prepend this string to prefixes to avoid storage problems
# leave blank to store event and data files in the running directory
SCRTCH=
# set the following variable equal to the list of object files that
# you need when using HERWIG (for analysis purposes, for example)
HWUTI="mcatnlo_hwannr.o"
# set the following variable equal to the name of the version of
# HERWIG that you use
HERWIGVER="herwig6510.o"
# set the following variable equal to the name of the directory where
# the PDF grid files are stored. Effective only if PDFLIBRARY=THISLIB
PDFPATH="/home/frixione/PDFgrids/"
# set the following variable equal to the name of the directory where
# the local version of LHAPDF is installed. We assume that the library,
# PDF sets, and configuration script are located in lib/,
# share/lhapdf/PDFsets/, and bin/ respectively
LHAPATH="/home/pcl225/nisius/MCatNLO/LHPDF5-4-1/lhapdf-5.4.1"
# set LHAOFL=FREEZE to freeze PDFs from LHAPDF at the boundaries,
# =EXTRAPOLATE otherwise. This variable is related to LHAPARM(18)
LHAOFL=FREEZE
# set the following variable equal to the names of the libraries which
# need be linked. Library names are separated by white spaces.
# Note: LHAPDF is a special case, and must not be included here
EXTRALIBS=
# set the following variable equal to the paths to the libraries which
# need be linked. Library paths are separated by white spaces.
# Note: LHAPDF is a special case, and must not be included here
EXTRAPATHS=
# set the following variable equal to the paths to the directories which
# contain header files needed by C++ files. Directory names are separated
# by white spaces

INCLUDEPATHS=

#
#
# NOW LOAD THE SCRIPTS: DO NOT REMOVE THESE LINES
thisdir='pwd'
   . $thisdir/MCatNLO.Script
#
#
#
#
#
# HERE, WRITE THE NAME OF THE SHELL FUNCTION THAT YOU NEED TO
# EXECUTE CHOOSING AMONG (ONLY ONE AT A TIME):
#
#      runMCatNLO    runNLO    runMC    compileNLO    compileMC
#
# THEIR MEANINGS ARE DESCRIBED IN WHAT FOLLOWS
#
#
# the following compiles and runs both the NLO and MC codes
runMCatNLO

# the following compiles and runs the NLO only (thus, the event file
# is written, but not read by HERWIG)
runNLO

# the following compiles and runs the MC only (thus, the event file must
# be already present, otherwise the program crashes)
runMC

# the following compiles NLO code
compileNLO

# the following compiles MC code
compileMC

runMCatNLO
C. Selection Efficiencies for the Parton Level Samples

In this appendix the selection efficiencies for the various configurations of the kinematic fit will be given for the following three parton level dataset states: The first sample is the full smeared sample without the events which exhibit an extra parton in the final state. The second sample with additional strict phase space requirements as described in Section 2.3. In the last sample an additional energy loss was simulated for the full smeared sample as described in Section 4.1.2. The numbers for the full sample can be found in Table 4.1 in Section 4.1.6.

Table C.1.: Purities and selection efficiencies for the parton level sample without the events with additional partons in the final state. The numbers are given for both $\nu$ implementations (T) in combination with the different enhancements. The abbreviation KF denotes the kinematic fit, b stands for b jet identification, $P$ for a $P(\chi^2)$ limit of 0.15, and $m_t$ for the top quark mass range limitation of 170 $\pm$ 30 GeV. The efficiency of the kinematic fit is the product of the triplet purity times the fraction of selected events (SE). All numbers are in % and all uncertainties are 0.1%.

<table>
<thead>
<tr>
<th>T</th>
<th>b</th>
<th>P</th>
<th>$m_t$</th>
<th>Kinematic Fit</th>
<th>$p_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{\nu,z}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>Correct</td>
<td>Triplet</td>
<td>Efficiency</td>
<td>SE</td>
<td>Triplet</td>
</tr>
<tr>
<td>98.7</td>
<td>55.6</td>
<td>70.0</td>
<td>69.1</td>
<td>100.0</td>
<td>39.4</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99.0</td>
<td>55.2</td>
<td>69.9</td>
<td>69.2</td>
<td>100.0</td>
<td>39.4</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>92.4</td>
<td>56.8</td>
<td>70.8</td>
<td>65.4</td>
<td>100.0</td>
<td>39.4</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>91.5</td>
<td>56.6</td>
<td>71.0</td>
<td>64.9</td>
<td>100.0</td>
<td>39.4</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>82.2</td>
<td>66.1</td>
<td>82.7</td>
<td>68.0</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>82.7</td>
<td>65.3</td>
<td>82.2</td>
<td>68.0</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>x</td>
<td>77.4</td>
<td>67.1</td>
<td>83.2</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>x</td>
<td>77.3</td>
<td>66.2</td>
<td>82.7</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>97.2</td>
<td>70.2</td>
<td>76.4</td>
<td>64.2</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>97.3</td>
<td>69.9</td>
<td>76.2</td>
<td>74.2</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>x</td>
<td>89.0</td>
<td>71.1</td>
<td>77.2</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>x</td>
<td>87.8</td>
<td>71.1</td>
<td>77.4</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>x</td>
<td>83.6</td>
<td>80.5</td>
<td>87.4</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>x</td>
<td>83.9</td>
<td>79.9</td>
<td>86.9</td>
</tr>
<tr>
<td>$p_{\nu,z}$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>77.1</td>
<td>81.1</td>
</tr>
<tr>
<td>$\eta_{\nu}$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>76.7</td>
<td>80.6</td>
</tr>
</tbody>
</table>
Table C.2.: Same as in Table C.1, but for the parton level sample with strict phase space requirements. All uncertainties are 0.2 % or smaller.

<table>
<thead>
<tr>
<th>T</th>
<th>b</th>
<th>P</th>
<th>m_4</th>
<th>Kinematic Fit</th>
<th>p_{tmax}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SE</td>
<td>Correct</td>
<td>Triplet</td>
<td>Efficiency</td>
<td>SE</td>
</tr>
<tr>
<td>$p_{\nu z}$</td>
<td>98.8</td>
<td>55.3</td>
<td>69.8</td>
<td>68.9</td>
<td>100.0</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>99.1</td>
<td>55.1</td>
<td>69.9</td>
<td>69.3</td>
<td>100.0</td>
</tr>
<tr>
<td>$p_{\nu z}$</td>
<td>x</td>
<td>92.2</td>
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Table C.3.: Same as in Table C.1, but for the parton level sample with simulated energy loss. All uncertainties are 0.1 % or smaller.

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D. Selection Efficiencies for the Full Simulation Samples

In this appendix the selection efficiencies for the various configurations of the kinematic fit will be given for sample 4Ja and 5Ja.

Table D.1.: Purities and selection efficiencies for the matched full simulation sample without strict phase space requirements in the four jet case (sample 4Jam). The numbers are given for both $\nu$ implementations (T) in combination with the different enhancements. The abbreviation KF denotes the kinematic fit, b stands for b jet identification, $P$ for a $P(\chi^2)$ limit of 0.15, and $m_t$ for the top quark mass range limitation of 170 ± 30 GeV. The efficiency of the kinematic fit is the product of the triplet purity times the fraction of selected events (SE). All numbers are in % and all uncertainties are between 0.4 % and 0.7 %.

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